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Beyond Estimation

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אשרי אדם מצא חכמה ואדם יפיק תבונה:[2]

Proverbs [3:13]

ζητῶ γὰϱ τὴν ἀλήθειαν, ὑφ' οὐδεὶς πώποτε ἐβλάβη.[3]

Marcus Aurelius (121-180), Meditations

AMICUS PLATO SED MAGIS AMICA VERITAS.[4]

Roman Proverb

Introduction

My last public lecture was more than four years ago. Because this may be my last public lecture for an equally long time, I won't waste your time with mostly technical matters. I will try also to keep equations to a minimum. I wish, in fact, to address topics that don't usually find their way into the keynote addresses of events such as this. I will talk about some informative subjects, some humorous ones, and also about some serious ones. I hope you won't be disappointed.

I would like to begin by telling you why I feel the development of Attitude Estimation is intrinsically different from that of the other areas of Astronautics. I would like to talk also about the crisis in Engineering Education, about the unfortunate role of prejudice in Engineering and Science, and about the specific role it has played in my career. I would like to tell you what I think is the true role of the Arts in Engineering; and about my efforts to bring humor to Astronautics. Lastly, I would like to confess to you

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[2]Happy is the man who findeth wisdom, and the man that getteth understanding.

[3]For I seek the truth, by which no man was ever harmed.

[4]Plato is my friend but truth even more so.

what I feel is my proudest accomplishment in Astronautics. You may be surprised at what a humble thing I treasure. Without question my talk will be a mixed bag and, unlike my career in Astronautics, without a strong focus. It will be rather a collection of half-a-dozen or so loosely connected little essays. In a word, or rather in two words, I wish to talk about the part of my "business" which is *beyond estimation*.

I do not insist that you agree with me on any topic I address. But I do insist on speaking honestly and forthrightly. Honesty is a trait often praised but seldom honored.[5] I hope that will not be the case here. All of the opinions I express are, of course, my own and not necessarily those of either the American Astronautical Society or the University at Buffalo.

Readers of my papers may have noted my increasing love of quotations at the head of my papers, especially quotations in ancient languages. I claim no special erudition, but I think that the ancients somehow knew how to say things with fewer words and with more force than we today in our more humble languages. I have always felt that such quotations anchor my papers in the real world. This practice actually is not new for me. My doctoral dissertation, written 35 years ago, contained a Latin quote by Virgil and a *rubaï* by Omar Khayyam (in the Fitzgerald translation). As you will learn from a later section of this talk, I think there is real value in puzzling out the meaning of a quotation in the original language and becoming aware of the different structure of the original expression.

The present text has evolved a great deal from the presentation at the symposium. All of the sections of the symposium presentation are here, but its present form is much more tightly written, more balanced, and better reasoned. The version of the keynote address to appear in *The Journal of the Astronautical Sciences* will be different also, sharing, perhaps, only half of its material with the present version, the result of the more restricted focus of the Journal and my original intent that the journal version have a different emphasis. Readers of the journal version will find there a more detailed historical section and a new section, which I had earlier deleted from the keynote address, because it had become too long, and even a somewhat different title. I hope that readers will not be too disturbed by this scribal schizophrenia.

The Youngest Quadrant

Slow are the beginnings of Philosophy.

Henry David Thoreau (1817–1862)

Les commencements ont des charmes inexprimables.[6]

Jean-Baptiste Poquelin (Molière) (1622–1673)

When I first began doing attitude estimation in the late 1970s, the field was in a very underdeveloped state. This is not surprising, since the space age, which began with the

[5] "Probitas laudatur et alget," Juvenal (fl. 127 CE).

[6] Beginnings have inexpressible charms.

Table 1. Founders of Astronautics

	Dynamics	Estimation
Orbit	Newton (1642–1727) Lagrange (1736–1813) Hamilton (1805–1865) Einstein (1874–1955)	Kepler (1571–1630) Lagrange (1736–1813) Gauss (1777–1855)
Attitude	Euler (1707–1783) Cayley (1821–1895)	(gone fishin')

launch of Sputnik (October 4, 1957), was not quite two decades old. Clearly, Spacecraft Attitude Estimation as a field was younger still. Even more interesting is the fact that Attitude Estimation was largely undeveloped even without the qualifier "spacecraft."

If I examine the four quadrants of Astronautics, which I divide as either Dynamics or Estimation and as either Orbit or Attitude, and look at who and when the early important work was done in each quadrant,[7] I find a huge blank in my own quadrant. (see Table 1)

My table is simplistic, to be sure. Newton, Kepler and Euler certainly had their antecedents, and the development of Mechanics by Newton was the culmination of a long and continuous process [3]. For orbit estimation we might even point to Claudius Ptolemy (87?–150?) or, still earlier, to Hipparchos of Nicaia (190?–125? BCE), but that would really stretch the point, because these celebrated scholars could not estimate celestial distances. The message which I wish to convey is that while three of the quadrants are populated by several intellectual giants of old and can boast of 250 years or more of development, the south-east quadrant is sadly deserted. There were apparently no eighteenth- or nineteenth-century contributors to attitude estimation of even modest calibre. In May 1977, when I formally entered the Astronautics community, there was one lone young university professor, John Junkins, who had just begun to publish some work on attitude estimation, but the field would not become his focus. (Fortunately for me, else there would have been nothing for me to do.) There were no courses taught on the subject nor any books, monographs, or even many journal articles. Obviously, the situation is much better now.

While Spacecraft Attitude Estimation had made only very slow progress, Spacecraft Attitude Determination, in which I include instrumentation as well as data processing algorithms, was very much alive. By the early 1970s the Apollo Mission was over and

[7]Despite his enormous contribution to Mechanics in general, the reader may be surprised that I do not cite Hamilton as a founding father of Attitude Dynamics. This is because he never wished to understand the connection between quaternions and rotations [1, 2], He seems to have been rather pigheaded about it as well.

we had sent spacecraft to Venus, Mars, Jupiter and Saturn. Hundreds of spacecraft had been placed in Earth orbit. The great romantic triumphs of the space age had been accomplished. Only the very earliest of these had taken place without a three-axis attitude determination system (and the entire movie-going public knows that the Apollo-13 attitude determination and control system at the time of the final orbit adjustment may have left much to be desired). The mathematics of mission attitude determination, however, had not advanced as quickly as the instrumentation. As attitude sensors were becoming more accurate and attitude computations were needed with greater frequency, this difference needed to be addressed.

There were obviously some important discoveries in Attitude Estimation early in the space age by excellent astronauticists even if clearly inferior in their gifts to the demigods appearing in the table, people like Harold Black (for TRIAD[8]) [6], Grace Wahba (for her famous problem, posed while she was still a graduate student)[9] [7], and James Farrell [8] for the first articles on the attitude Kalman filter. These people are all still alive. However, unless you do work in attitude estimation, you may never have heard of them. A landmark in my quadrant was the *Symposium on Spacecraft Attitude Determination* [9] held at the Aerospace Corporation in 1969, at which a lot of Kalman filter work was presented, some of which, especially the Kalman filter of Toda, Heiss and Schlee [10], in highly "filtered" form, found its way into Ref. 11, as did the PADS algorithm of Iwens and Farrenkopf [12], which followed soon after. At the level of the celestials in Table 1, however, my quadrant will likely always remain unfilled.

There is, in fact, a good historical reason for the emptiness of my quadrant. During the great classical period of Mechanics and Astronomy, there was simply no great problem in Attitude Estimation waiting to be solved. There was no Kepler problem to entice a Newton, no problem of the orbit of Ceres for a Gauss of attitude estimation, no spinning top for an Euler or a Cayley. The closest one came to Attitude Estimation in those halcyon days was the problem of the libration of the moon, hardly a great watershed of Physics, which must have required some measurement of the deviation of the orientation of the moon from its mean orientation relative to the Earth-moon line. But this could not really be called attitude estimation in the way that Lagrange and Gauss truly did orbit estimation. Thus, there is no great hoary founder of attitude estimation. My field would have to depend on rather more puny personages for its advancement, people like you and me.

Thus, those of us who currently inhabit the youngest quadrant have had to do it all ourselves, benefiting, naturally, from analogous work done in the other quadrants but with much less talent on our part. The bleakness of our panorama three decades ago can

[8]A bit of history: I am responsible for the name TRIAD. I first encountered this algorithm at the outset of my career in space via Wertz' book [4], where it is called the "algebraic method" (in contrast to the "geometric" method), a name I never liked. When I was writing my first journal article in Engineering [5], which would treat QUEST and the algebraic method, I went looking for the origin of the "algebraic method." The earliest use I could find at the time was a company report from IBM Federal Systems Division, giving the functional specification for a spacecraft attitude determination system called "Tri-Axial Attitude Determination System" with acronym TRIAD. So I renamed the algebraic method the TRIAD algorithm, which I liked also for the additional reason that the algorithm constructed the attitude matrix via orthonormal triads. The name, however, was in capital letters, because it was an acronym. Only later did I discover Harold's paper, which predated the TRIAD software, and made that my primary reference.

[9]Note that my Brouwer lecture, as presented in 2001 [2], offered incorrect information for Dr. Wahba's professional location at the time she posed her famous problem.

be seen in the justly celebrated book of James R. Wertz, *Spacecraft Attitude Determination and Control* [4], written by 35 employees of the Attitude Systems Operation of the Computer Sciences Corporation in Silver Spring, Maryland, which has been an essential reference for Spacecraft Attitude Estimation since its first appearance in September 1978. What attitude estimation can be found in its nearly 900 pages, however, is almost entirely on spin-axis attitude estimation, which, for me, is not really attitude estimation at all. F. Landis Markley, one of the major contributors to the book, has a section of not quite eleven pages on the attitude representations [13], which served the attitude community well for more than a decade and twenty-five years later is still sufficient reference for most of that community. Gerald M. Lerner has a section on three-axis attitude determination [14] (only eight pages) in which he gives a succinct account of the TRIAD algorithm (algebraic method) and the first archival account of Davenport's q-method. And that's it, except for a few pages on least-square attitude estimation later in the book [15]. The chapter in Wertz on "State Estimation Attitude Determination Methods" contains no three-axis attitude estimation at all. This deëmphasis of three-axis attitude determination is a reflection of the particular experience of the book's authors, who wrote for the most part only about the missions they had supported for NASA Goddard Space Flight Center in the 1970s, most of which had employed spinning spacecraft.[10] Three-axis attitude determination, however, went back more than a decade before Wertz' book, although little of it (or of any part of attitude estimation) could be found in the open literature. This situation was already changing when I joined CSC. During my tenure there, most of it after the book had appeared, nearly all of the spacecraft I supported required the determination of three-axis attitude.

Despite this deëmphasis of three-axis attitude, the book was a solitary light shining in the darkness of the youngest quadrant, containing much information on sensors, data analysis, the essentials of three-axis attitude estimation, and, of course, a lot of spin-axis attitude estimation. In it you will find a reference to Markley's work on Attitude Control but none of his innovations in Attitude Estimation. They were soon to come. For the work of this writer you will find only a single sentence at the end of Lerner's section on three-axis attitude determination: "Variations on the q-method which avoid the necessity for computing eigenvectors are described by Shuster [1978a, 1978b]." This, of course, was the QUEST algorithm, my very first task in attitude estimation, executed with much clumsiness and trepidation [16], and at this time still at a fairly primitive stage. Very likely, it would not have received mention at all had I not been working at CSC (and Jerry Lerner not been my close friend of more than a dozen years). In any event, my first contribution to Attitude Estimation wasn't worth more than a single sentence at the time.

It is important to stress the importance of the Attitude Systems Operation at the Computer Sciences Corporation for the development of Spacecraft Attitude Estimation in the 1970s. In an era when a great many Ph.D. scientists and mathematicians were looking for jobs outside academia, CSC discovered that these highly-educated but misdirected individuals made good analyst-programmers. The number of Ph.D.'s in the 50-man Attitude Analysis Department was very large, and, when I arrived for my interview there in February 1977, perhaps even half. The situation for the Orbit Operation was similar, leading, perhaps, to the largest group of astronautists devoted to

[10]The impetus for the book, in fact, had been to create a handbook for NASA/GSFC attitude support.

the development and execution of algorithms for attitude and orbit estimation anywhere in the world.

Ph.D. physicists are raised in a culture of teaching, research and publication, and those inclinations do not necessarily desert them when they enter industry. With more than a half-dozen missions in active development at any one time at NASA Goddard Space Flight Center, the principal customer of my CSC division, and the constant need for improved attitude estimation methods, it was inevitable that CSC would be drawn into attitude research. The job also required that CSC analysts be involved in software development and in launch and early mission operations as well. Thus, CSC provided an excellent practical education for its attitude analysts and a research-minded environment as well. The results you know.

Following the publication of *Spacecraft Attitude Determination and Control*, the situation changed dramatically, in no small part because of that book. For more than a quarter century since the appearance of *Spacecraft Attitude Determination and Control*, Landis Markley and I have made a great effort in our journal articles to extend the material in *SADC*, especially for three-axis attitude estimation and the Kalman filter applied to attitude estimation, as well as for inflight estimation of attitude sensor alignment and calibration parameters. Over the past decade or so the degree of activity on spacecraft attitude determination has increased greatly. It is now a mature field and an important component of virtually every professional meeting on Astronautics. The number of journal articles on the subject now is substantial as is the number of university professors who consider Spacecraft Attitude Estimation a fit subject for research. When I entered the field in 1977, with no background beyond a few weeks of rigid-body mechanics while a Physics student, there was little to learn, and one learned it all very quickly. Now, almost 30 years later, I no longer try to keep up with every new development.

This is about all that one can write on the early history of Spacecraft Attitude Estimation. Attitude Estimation is in many ways a street waif, of ambiguous origin and growing up much too quickly. And yet it has turned into a handsome adult, highly sophisticated and a joy to behold. It is a joy for me to see here today so many of the people who have made attitude estimation into such a vigorous field.

This joy is not unalloyed. Attitude Estimation faces new and difficult challenges. Possibly, my quadrant is the most demanding mathematically of the four. Not only must it deal with the complicated nonlinear equations of attitude dynamics, but it must deal with uncertain dynamics and measurements as well. At the same time, university graduates seem less equipped now than a generation ago to learn the mathematical disciplines necessary for research in Attitude Estimation. Current trends suggest that things will get much worse before they get better. This problem and its causes will permeate much of this talk.

The Miseducation of American Engineers

SCHOLAE NON VITAE DISCIMUS.[11]

Lucius Annaeus Seneca (4 BCE–65 CE)

SCHOLAE NON VITAE DOCEMUS.[12]

Malcolm David Shuster (b. 1943 CE)

It is no secret that Engineering students nowadays are not as solidly educated on the average as were those of my generation (the 1960s), but the problem has reached a level that no one would have imagined even only two decades ago. The problem is less apparent at the most prestigious Engineering colleges and does not exist for the very best Engineering students, the ones who will eventually occupy faculty positions, the students of whom Gibbon[13] once said "the power of instruction is seldom of much efficacy except in those happy dispositions where it is almost superfluous." Rather, it is a serious problem for the more typical Engineering student, who will eventually work in industry and government. Our very best Engineering students are better prepared than those of my generation, but preparation of the average Engineering student appears to have fallen dramatically. Even the Aerospace Engineering department at one prestigious Engineering school has admitted the decreased undergraduate performance since the 1960s (and the inflation of grades).

We are confronted today (at least from my personal experience) with new entering Engineering undergraduates (with high math SAT scores) half of whom cannot sketch simple trig functions, with seniors (with GPAs above 3.9) who have long forgotten that $\log ab = \log a + \log b$, and with advanced graduate students who cannot write the formula for matrix multiplication in terms of elements, or who do not know that if $y = \log_{10} x$, then $x = 10^y$. I have seen all of these things first hand. For too many of our Engineering students "log," "sin" and "cos" are simply buttons on a pocket calculator, and theoretical understanding has been replaced by the ability to code in Matlab®. Colleagues from many other universities have similar complaints.

As a recent university professor I was a contributor to this problem, tailoring my exams to what I thought the students could answer rather than what I thought they should know and, even then, sometimes giving higher grades than were justified by the exam results. Worse, as a senior engineer in industry, my longer and more significant career, I knew that I could not possibly hire most of the Engineering students I taught in the 1990s. I could not expect them to learn the trade well enough to be useful and I could not trust their work.

The obvious solution is to raise standards to what they were three or four decades ago. That, unfortunately, would have the result of decreasing university enrollments in Engineering and, consequently, university revenues. For state universities it would also create dissent in state legislatures and in the general population, which would see its children shut out from a better future. The unfortunate compromise has been to

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[11] We learn for school and not for life.

[12] We teach for school and not for life.

[13] Edward Gibbon (1737-1794) was the author of the *Decline and Fall of the Roman Empire*. His statement is also quoted by Richard P. Feynman in the prefaces to his *Feynman Lectures in Physics*.

lower the standards of university textbooks and courses and to inflate grades.[14] We have been able to carry the charade only so far. Engineering departments learned some time ago that they must import most of their graduate students from abroad, because the well-qualified portion of the American product we produce ourselves is insufficient. We still graduate many fine American-born Engineering students, but not enough.

The problem cannot be solved within the universities alone. Our K-12 educational system, as has been generally recognized, has fallen prey to the same diseases as university education.[15] Too often our freshmen not only lack the skills they will need for Engineering studies, they have never developed the discipline which serious study requires, a result, I believe, of the decades-long K-12 policy of rewarding students for participation rather than for achievement. In this way many students who might otherwise have become successful engineers become instead incapable of profiting from the opportunity of an Engineering education. We cannot raise university standards without also raising those of K-12. Vested interests in that arena, however, make change difficult as well, as do economic factors, which also hobble the universities but in a different way. It is a problem that resists solution and can only get worse, but it is a problem that demands solution . . . soon. We cannot continue simply to lower our expectations of student performance.

There may be a biological factor as well. University enrollments as a fraction of the total college-age population are double what they were 40 years ago, with the result that the *average* IQ level of our university student bodies must have fallen.[16] There's no obvious reason to believe that this diminution of (monochronic) IQ is not true also for that segment of our university student bodies which wishes to major in Engineering. Some of my colleagues contend that changing student demographics may have mitigated any decrease in overall undergraduate (monochronic) IQ. That argument might seem reasonable for much smaller university enrollments than are the case, but not for actual enrollments as large as 25 percent (1970) and 50 percent (1998) of the total college-age population of the United States.[17]

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[14]There is no better proof of the pandemic nature of the lowered performance of our undergraduate Engineering students than the "dumbing down" of our undergraduate textbooks.

[15]I recently read that studies have shown that learning the multiplication tables by heart leads to a more successful academic career. The great shock to me was not that students no longer know the multiplication tables, nor that a study of this kind was needed, but that ignorance of the multiplication tables was not by itself considered to be academic failure. How much longer will it be before improvements in voice recognition software permit the equal abandonment of the other two Rs?

[16]Here we must distinguish *monochronic* IQ scores, normed (mean = 100, standard deviation = 15 (Wechsler) or 16 (Stanford-Binet) for the year of the test, and *dichronic* IQ scores, normed to a the distribution of a different year.

[17]Legacy preferment and discrimination may affect the student's university of choice but not whether he goes to the university at all. The dramatic increase in minority enrollment reflects for the most part the dramatic increase of minorities of non-European origin as a fraction of the general population. Minority enrollment has likely grown relative to minority population, but the total figure even today is only 30 percent of university enrollment. Female enrollment has increased from 42 percent to 56 percent since 1970, a change in the composition of only 14 percent of university enrollment, and the new 14 percent of female enrollment is unlikely to be of higher IQ than the previous 42 percent. These factors together are certainly significant, but fall far short of balancing the difference in IQ of the first and second quarters of the U.S. population. The IQ has many shortcomings, and I am hesitant to rely on it too strongly. Nonetheless, monochronic IQ has been shown to be a significant indicator (in the aggregate) of academic and professional success.

As a counter argument we may mention the well-known Flynn Effect, the increase of dichronic IQ scores

There are, clearly, many factors adversely affecting Engineering education: inadequate preparation and anti-achievement conditioning in the public schools, overpopulation of the university student bodies, and our lowered expectations. There is also the factor of the attraction of newer "hard" disciplines, especially Computer Science, which competes for the same pool of students as does Engineering. National degree-accrediting agencies also exacerbate the problem by demanding that more advanced topics be squeezed into the narrowly allowed confines of the undergraduate programs, thereby forcing out more important basic courses. Equally important may be a deëmphasis of teaching in the Engineering faculties. University tenure decisions are based far more on the candidate's demonstrated skill in acquiring research funding than on the quality of his teaching. In addition, the time expended in applying for increasingly smaller, shorter, and more intensely competed research grants limits the time that even highly dedicated teachers can devote to class preparation. To the regret of many, the business of colleges of Engineering has become business.

A factor swelling the universities with poorly qualified students is the overemphasis of university education. Where once a high-school diploma was sufficient, as in office work, many employers now insist on a university degree. If the universities come to make it possible for every high-school graduate to receive a university degree, the inevitable consequence will be that a university degree will only have the value of the present-day high-school diploma and not even that of a high-school diploma of forty years ago. None of these factors, however, should justify lower standards. Nor can they excuse the fact that too many Engineering sophomores with math SAT scores in excess of 700 cannot solve simple problems in high-school Algebra and Trigonometry. Flynn has found a similar lack of connection in recent decades between the rise in IQ among university students and their ever decreasing level of academic achievement. He attributes this phenomenon to social factors, although thorough analyses do not yet exist. Thus, it may be anti-achievement conditioning by K-12 and by the American culture as a whole which has had the greatest effect, and this will be compounded by the swelling of university enrollment, since the bottom half (by IQ) of that enrollment is likely to even less driven by achievement than the top half. A biological factor then cannot be denied. Even if this is true, however, it is one thing to understand the origin of a problem and another to correct it.

We have been able to compensate for our dilemma only by reliance on foreign intellect.[18] Twenty percent of the four million engineers and scientists in this country

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with time [17]. In the U.S. Flynn has been able to document a steady rise in dichronic IQ scores normed to the present population of about 3 points per decade from 1932 to 1978 [18], a rate of increase similar to what he has found for many European countries. The rise (in Europe, at least) is mostly attributable to environmental effects, particularly nutrition in the post-war years, although the interplay between genetic and environmental factors is complex [17]. (In the U.S. analysis of new data by Flynn, some of it unpublished, pushes the steady rise of dichronic IQ up to 2002. What this means, as Flynn points out [18], is uncertain. Can we believe that the average IQ in 1902, normed to today's IQ levels, was only 70?) Improvements in education are also an important factor in the increase of dichronic IQ scores. One hundred years ago, Algebra and Trigonometry were college subjects. It must also be pointed out that as society becomes more complex technologically and socially, the need for higher *absolute* intelligence must also increase, making the monochronic IQ scores the more relevant. In any event, the biological/environmental factor in our students' capabilities is very complex (see related comments in the next section).

[18]Hardly a new phenomenon, if much increased today. The comedian Bob Hope, following the launch of Sputnik, quipped: "This just goes to show that their German scientists are better than our German scientists."

are foreign-born, as is the *majority* of new Ph. D.'s in Science and Engineering. Not surprisingly, it is our foreign-born and first-generation high-school students who carry off most of the academic and science prizes. Engineering and Science departments in this country have always attracted foreign students, but in the 1960s, when I was a graduate student, foreign graduate students were very far from constituting a majority. Those figures are likely to increase in the short term. However, as high-tech industry in Asia increases, so will the quality of Engineering education there and the ability of Asian universities to retain their best Engineering students. The many Indian Institutes of Technology are already the equal of our best Engineering schools. We already find ourselves less able to attract the best Asian students to our graduate schools in Engineering, in no small part due to the shrinking of available research funds and the reluctance of funding agencies to allow their funds to support the activities of non-citizens. This situation will be exacerbated by the acculturation of the children of our present new immigrants, who will gradually succumb to the anti-intellectual American norm.[19] In any event, because of concerns for national security and technology transfer, foreign-born engineers cannot compensate for the diminution of well-qualified applicants for Astronautics positions in government and industry. We will begin to see real problems there sooner than in academia.

We may even come someday to see a brain drain of American engineers of European origin to Asia, especially to India, where English is the *lingua franca* of higher education and the educated.[20] A reverse brain drain of Asian-born engineers has already begun. Perhaps, would-be American Engineering students will someday take special classes in order to qualify for admission to Asian Engineering schools. We are already witnessing the exportation of Engineering design work to Asia. Much high-tech manufacturing has already departed from our shores, admittedly more a K-12 problem and dependent also on economic factors. Nonetheless, the departure of supporting Engineering work can only follow. We may have already lost the leadership of some key space technologies, such as star trackers.

The deficiencies of our K-12 education are already having an adverse effect on the location of new foreign manufacturing plants in this country. Canada, with its national health-care program and a better educated workforce is proving more attractive. Our national space program is dependent on the national economy, and the continued loss of manufacturing cannot be beneficial.

In our undergraduate Engineering programs we must concentrate more on making certain that our students have acquired a solid basis in Engineering rather than a broader range of Engineering topics treated only superficially. It is important that our students not be unable to see the forest for the trees, but they cannot understand the forest if they have only a limited understanding of trees. We cannot continue to graduate students who are woefully deficient in the basic tools of Engineering but believe wrongly that they have the "big picture." The future of Engineering in this country, Astronautics included, may be in peril. In my area of specialization, and in many others, there are already clear symptoms.

[19]If we believe Alexis de Toqueville (1805–1859), American anti-intellectualism goes back at least to the early nineteenth century.

[20]Let us not forget that before World War II many young American scientists and physicians went to Europe for further training.

Prejudice in Science and Engineering

NIHIL AGENDO HOMINES MALE AGERE DISCIUNT.[21]

M. Porcius Cato (214–149 BCE)

Je mehr ich von den Menschen sehe, um so lieber habe ich meinen Hund.[22]

Frederick the Great (1740–1786)

Religious, racial, ethnic, and gender prejudice are horrible things. They poison both the person who expresses prejudice and the person about whom it is expressed. We have made great strides in all of these areas, but they have not yet disappeared completely. Diversity in some cases has become the cover name for discrimination, just as it has been the watchword for eliminating discrimination. I would like to examine the effects of these prejudices on Engineering and, in particular, on this symposium.

A great step backward, I think, is the new essay question of the SAT. This will be a disadvantage to foreign-born students and to those American-born students who grow up in households where standard English is not spoken. The likelihood that many of the examiners evaluating these exams, whose numbers must be astronomical, will not be affected by cultural cues or a foreigner's or minority's particular spelling, grammar, or idiom rather than by the student's ability to write coherently and clearly, is very small. Unfortunately, these are the students from whom we often draw our best Engineering undergraduates. I think the essay question is a big mistake. Multiple choice exams, which can be evaluated by computer, have many deficiencies, but they are largely insensitive to the ethnic or national identities of those taking the exam. The essay question itself is not discriminatory, but its very nature facilitates discrimination. To think that the SAT essay question might somehow improve the writing skills of our students seems very naïve.

The elimination of gender discrimination in Engineering seems to be making great progress. When I entered MIT in 1961, a cultural epoch as distant from the present as from the ante-bellum South, my freshmen class had fewer than three percent women; the class entering in 2005 will have 49 percent.[23] Religious intolerance, unfortunately, seems to be on the upswing, but it is unlikely to affect Engineering as much as it will our quotidian existence, unless we become forced to teach religious doctrine as science in our public schools. We have made great progress in racial discrimination also, but we are far from finished there, and I am not even certain that we have addressed all of the important factors. The SAT essay question, as I have said, will certainly open the door to random acts of racial or ethnic discrimination in university admissions, with particular harm to potential engineers.

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[21] By doing nothing they teach men to do evil.

[22] The more I see of people, the more I love my dog.

[23] While some schools, like MIT and Princeton, may have achieved gender parity in their entering Engineering undergraduates, the national fraction of female Engineering undergraduates in all years is closer to 20 percent, and only about 15 percent of Engineering Ph. D.s currently are granted to women. Eventually, we may achieve gender parity at all academic levels in Engineering. The process is a long one, however, the length of a faculty career.

Racial prejudice is a very dangerous thing to discuss in a public forum, so, of course, I will discuss it. Racial prejudice is not as big a problem as it used to be, but I think it is still there, and I don't think that we have done everything we could or should to fix it. Legislation, in particular, has not done everything it could, and severe inequalities still remain. I wish to talk about only one small part of those inequalities, which are true, I think, of all our minority underclasses, whether they be the urban or the rural poor. It crosses color lines, though surely some racial groups are much worse off than others.

We all know that these minority underclasses are underrepresented in our universities, and the immediate cause is that they test more poorly than more affluent classes. The evidence is incontrovertible, and it is found not only in SAT scores but also in IQ test scores. Some special interest groups contend rightly that these minority groups have very poor schools and that the deficiencies of K-12 education for these disadvantaged segments of our society is the largest part of the problem. There is much truth in this statement, but there are other social factors besides public education which control the development of human intellect.[24].

A very large part of cerebral development takes place during the prenatal, neonatal and early-childhood development periods. If the child doesn't receive adequate nutrition in those periods or is forced to ingest harmful substances, the intellectual development of the child will be impaired and not all of this impairment can necessarily be compensated by the improvement of nutrition in later development. Studies have shown that it is only prolonged severe malnutrition which results in permanent impairment of IQ. But for how much of the population is severe prolonged childhood malnutrition a problem? Surveys have shown that seven percent of American toddlers suffer from severe iron deficiency, and the rate of deficiency increases with age. This deficiency has been shown to result not only in IQ loss but in extreme antisocial behavior in adolescence. Around twenty percent of African-American and Mexican-American women of childbearing age suffer from iron deficiency, with obvious implications for prenatal development.

In addition to nutrition there is also the problem of intellectual stimulation. The quantity and quality of intellectual stimulation in the homes of the underclasses is likely, on the average, to be much less than those of the more affluent college-educated classes. The advantage of exposure to classical music, literature, and art at all levels of public education is well documented. Not all of this intellectual deficit can be compensated in later years, almost certainly not after the onset of puberty. I do not believe that genetics is an issue at all. The majority of studies show that IQ is largely hereditary, but social class is also largely hereditary, if not genetically.

The problem of racial and economic differences in scholastic performance cannot be solved overnight, because ultimately it is the disadvantaged minorities themselves which must participate in providing the proper nutrition and intellectual stimulation for their children. But we can certainly accelerate this process with greater attention to the nutritional needs of our underclasses and a greater investment in their schools and, especially, preschool and early-school outreach programs. The efficacy of such programs in raising IQ has oft been demonstrated. Until we do, the problem will continue to get worse, and a significant source of engineers will remain lost to us. We may never correct these differences completely. Even the New Testament affirms that the poor will

[24]See my comments on the Flynn Effect in footnote 14, especially Ref. 17, of the previous section.

always be with us (John [12:8]), but we can make things better, and we can give people greater hope.

Antisemitism, because it has both ethnic and religious components, must be examined separately. It seems to have disappeared almost completely in higher education.[25] In our middle-class popular culture it is even chic now to be Jewish. This may make it all the more surprising to learn that this Symposium might not have taken place had it not been for an occurrence, only thirty years ago, of antisemitism.

During my first year in Engineering, I was asked to apply for the position of U.S. representative to an international energy committee headquartered at one of our national laboratories. I applied for the position with alacrity—such was my dedication to Spacecraft Attitude Estimation at the time. I was an unusually suitable candidate. This opinion, apparently, was shared by the entire selection committee, which thought I was head and shoulders above every other applicant. Nonetheless, I wasn't offered the position. The reason, I was told later, was that one very senior member of the committee, someone with a lot of clout, acknowledged my high qualifications but objected to my appointment. The previous two representatives had been Jewish, he said, and it was time for a change. In 1978 this argument did not elicit the outrage it would today. So I stayed in Astronautics, probably much to my benefit.

Had I been offered the position, I certainly would have accepted. In the late 1970s shuttling around the great industrialized countries of the world and going to more exotic countries on fact-finding missions seemed like the good life to me. Much as I am an American in my heart, I find life in Europe, with its greater cultural diversity, more interesting, especially given my interest in languages. My stomach, on the other hand, is also deeply committed to the Third World.

My career in Astronautics would have been over almost before it had begun. My only publication in Astronautics in the open literature would have been a lone conference report that wasn't even on attitude estimation [19]. A somewhat incomplete QUEST would have existed only in a CSC company report. The Method of Sequential Rotations was in that report but not the hugely important TASTE test [16], nor the QUEST measurement model [5], nor the neat expression for the QUEST attitude-error covariance matrix [5]. We would still be calling the TRIAD algorithm the "algebraic method," and we wouldn't have a neat expression for the attitude-error covariance matrix for that algorithm either [5]. The Lefferts-Markley-Shuster Kalman filter paper [11], my further work on the Wahba problem, my works on alignment and calibration, and the 79-page survey paper on the attitude representations [20] would not have appeared. Needless to say, had I been offered that position, this symposium would not have taken place. My career in Astronautics and this symposium owe much to discrimination.

[25]A recent carefull researched book on antisemitism in college admisions is Jerome Karabel's *The Chosen: The Hidden History of Admission and Exclusion at Harvard, Yale, and Princeton*, Houghton Miflin 2005.

The Arts and Engineering

τῷ σοφῷ ξένον οὐδέν.[26]
Antisthenes (444–371 BCE)

QUALIS ARTIFEX PEREO.[27]
Nero Claudius Cæsar Augustus Germanicus, Imperator (37–69)

We are often told, almost always by non-engineers and non-scientists, that instruction in the Liberal Arts and the Fine Arts should be an important part of Engineering education. The position of the non-technical person is that a liberal education will make engineers better people and better able to understand and respond to the needs of society. American Engineering students, however, are unconvinced and greet such assertions with ridicule and scorn. Both sides are wrong. Exposure to the Liberal and the Fine Arts is important to the education of engineers, but not just to make them better members of society. Such exposure is important, because it will make them better engineers.

There have been many studies of the value of the Arts in primary and secondary education, particularly as a motivational device, and there have been even experimental studies that show that intense exposure to Art develops superior spatial-visual coordination and other basic skills. I'm not sure how much this will influence the generation of secondary-school students which asks, like the illiterate dancing student in the 1980 Alan Parker film *Fame*, why he must read the book when he can watch the video. I have no interest here in Art as a motivational device or as an aid in neurological development, valuable though these aspects of Art education may be. What I wish to discuss are the premises that creation in Art and creation in Engineering have much in common, and that the study of Art in *tertiary* education demands creative participation on the part of the student, while the study of Engineering most often does not.

Contrary to the misconceptions of most non-technical people and, probably, of very many Engineering and Science students, Engineering, Science and Mathematics are not purely deductive disciplines. Deduction tells us nothing about how we will determine the assertions we wish to deduce or by what deductive path we will deduce them from basic principles. At a still higher level, deduction does not tell us how even to choose our basic principles. Deduction is important to our discipline, because we cannot accept a theoretical assertion which cannot be deduced from basic principles. An inability or unwillingness to carry out the deductive process fully and rigorously often leads to very bad research. I have seen this many times. But deduction is probably not the most important activity of Engineering research. The most important activity is *induction*, how we determine the things we want to prove (or discover or design or invent)[28] from observation, analogy and the magical element we call intuition. Unfortunately, we cannot teach induction, senior-year projects to the contrary. So we teach our Engineering and Science courses as deduction with an occasional dollop of historical and physical

[26] To a wise man nothing is foreign.

[27] What an artist dies in me. (Literally: what an artist I perish.)

[28] Stated in other words, the most important part of research is not finding the solutions but finding the problems.

motivation, and we give the misimpression to our students and to the world that ours is primarily a rational deductive discipline.[29]

The other misconception, of course, is that Science and Engineering deal with immutable truths, which also is not exactly the case. We believe that such truths exist, of course, but not that we necessarily know them. In Science and Engineering we never have a complete theory and often we have to revise our ideas as our experience increases. The majority of people in our world do not understand this. Far too large a fraction of our population, when a scientific idea turns out to require revision, thinks this justifies the belief in such absurdities as intelligent design, creationism or astrology. The true world of Science and Engineering is cloaked in ambiguity and doubt. Our job is to find the best approximation of truth amidst that ambiguity.

How do we make our students (and ourselves) better inducers and better able to deal with ambiguity? Here, I believe, the best laboratories are the Liberal Arts and the Fine Arts. Serious novels, poetry, paintings, sculpture, etc., are not quantitative, and their art comes in no small part from ambiguity. My claim is that the Liberal and Fine Arts accustom us to working amidst ambiguity and uncertainty. This is not to say that professors in the Liberal Arts or Fine Arts can *teach* us how to deal with ambiguity or how to be inductive. But they can expose us to many tantalizing examples of ambiguity, and to a lot of sensations which don't exist in the realm of Science and Engineering. As engineers and scientists, we generally brush off the Liberal and the Fine Arts, because they are not quantitative and lack any kind of repeatability, but this is insufficient justification for neglecting them.[30] The truths of the Arts are not quantitative, nor are they universal principles, but the experience of increasing our understanding through examination and reexamination is more accessible in the Arts than in Science and Engineering. The appreciation of a work of art is truly an induction experience. Regretfully, my efforts to cultivate greater contact of engineers with the Arts have yielded only bitter fruit. I always told my Engineering students that they would become better engineers if they would read a poem every week, especially a non-narrative poem. You can imagine their response.

When we confront our students with a work of art, whether it be a (non-narrative) poem, or a (non-representational) sculpture or painting, that experience requires a creative (and often inductive[31]) effort on their part, not the creative effort of the artist who created these works, but creative nonetheless. The student must somehow go beyond the superficial form in order to appreciate the work. It is not simply a bunch of words, notes, colors, lines, curves, and surfaces. An artistic painting or photograph is not the same as a snapshot from the beach. A novel or a poem is not the same as a newspaper article or a product description. A sculpture by Michelangelo is very definitely not the same as an atlas of domains over which the surface of the statue has been accurately represented by two-dimensional splines. Some students, whether would-be engineers or humanists, cannot "get" what a work of art is all about, and we

[29]For the largest segment of our successful Engineering graduates, it is even sufficient that they be good deducers, since their work will consist largely of the application of existing methods or in very repetitive experimentation.

[30]Actually, there are repeatable truths related to the Arts, which come from the application of Science to the Arts, as in the computer-aided statistical analysis of literary texts, but this is repeatability in Science rather than in Literature. What makes literature art is lost in the noise of such analyses.

[31]Creativity and inductiveness are not synonymous, although creativity in Engineering is usually inductive.

are unable to teach them how to "get" art. At best they learn a lot of vocabulary and how to recognize different periods or different artists, but they never truly experience art. They can only repeat what they have been told. The first person to say "your eyes are like the stars" was a poet. The first person to repeat it was trite. It is that way in Science and Engineering too. Perhaps many of our Engineering students and many of our liberal arts students are like this. Engineering and Science research, I claim, are the more difficult professions. In some ways, significant research in Science and Engineering, not simply repeating experiments for different substances or applying the same technique over and over again to different systems, is more demanding than "artistic creation," because one must be able to do two things well. Einstein, a poet of Physics, also played the violin and made amateur art movies. Few poets of the English language can solve Mathematics or Physics problems.

Art is not the only part of the Liberal and Fine Arts which has this benefit. I claim that so does the learning of a foreign language, especially a language very different from our own, whose vocabulary may have different semantic ranges and whose grammar may exhibit very different morphological and syntactic forms from our own. Languages most separated from ours either in space or in time (or both) are likely the best. The puzzling out of meaning in such languages is not a very different exercise from puzzling out the meaning of Engineering results, and, certainly, it is a constant exercise in induction. Listening attentively to serious music, reading serious literature seriously, examining works of pictorial or plastic art, or even watching foreign films, all have something to contribute to our education as engineers. Better still is to engage oneself in the creative side of the Arts, however poor the results.[32]

Most important of all, we must remember that Engineering research is a creative act, and creation is always an expression of the imagination. Following someone else's mathematical proof or the progression of equations in Engineering or Physics is not an expression of the imagination. Creating that proof or those equations for the first time *was*. (Your eyes are like the stars!) The closest we come in Engineering courses to helping our students learn induction is in their homework problems, but not if these problems are only substitution problems, or repetitions of the text, or overly guided connect-the-dots problems, which is the general case, especially in the more recent textbooks. If the problems do not require significant non-repetitive effort from the good students, then they are really of no value except to enhance short-term memory. Unfortunately, we opt too often for homework problems which do not require much thought, perhaps, because so few students can be expected to solve any other kind. As in the well-known sports maxim, the *art* of Engineering is learned in the *struggle* to get the answer, not in simply being shown it. The activity of research in Engineering has far more in common with artistic creation or even with the appreciation of a work of art than anything which we usually teach in our Engineering classes. If we share his mother tongue, we might learn far more of an Engineering graduate student's promise as a researcher by having him write a poem or a short story than by examining him on Fluid Dynamics or Finite Element Analysis.[33]

[32]My own pitiable experience has been that writing a short story feels rather like writing an article in Engineering or Physics, while writing a poem feels like proving a theorem in Mathematics.

[33]This is not to say, as one colleague has already chosen to misinterpret my remark, that we should not test out students on their competence in Engineering but only in their ability to write poetry.

I do not practice the science of research but the art of research. *Ars gratia artis mechanicae*, Art for the sake of Engineering, to modify the motto of MGM.

Humor in Astronautics

Der Humor ist die Maske der Weisheit.[34]

Friedrich Dürrenmatt (1921–1991)

There is not enough humor in Astronautics. I am not in favor of turning our esteemed journals into imitations of *Joe Miller's Jests*[35] or our professional meetings into a sequence of stand-up comedy skits, but we can do much more than we are doing now. Therefore, since 1992 I have dedicated myself to raising the comedic level[36] of our field. I have often been something of a clown in my conference presentations, but now, I decided, the time had come to make my humor archival. Of course, I needed to be subtle, lest my attempts at humor would be thwarted by the editors and reviewers.

My first essay in journal humor[37] occurred in the paper "Attitude Determination in Higher Dimensions" [21], in which I examined the nature of attitude determination in spaces of dimensionality $n > 3$. I showed that $n - 1$ direction measurements would be necessary to determine the attitude unambiguously and developed the n-dimensional analogue of the TRIAD algorithm. Given the nature of the research, some attempt at humor was irresistible. Thus, after admitting the impracticality of my research, I averred that "should the dimensionality of our world ever increase without notice, we will be all the better prepared." The copy editor of the *Journal of Guidance, Control and Dynamics* tried to turn my whimsy into something more staid, but I changed it back.

I had tasted blood.

For my publications after May 2001 I had a serious problem. I no longer had an employer. I did not wish to appear without any affiliation, so I invented the Acme Spacecraft Company, patterned after the company names which appeared in Chuck Jones' Roadrunner cartoons. Also, rather than calling myself President or CEO, I took the title "Director of Research," which seemed more imposing. The Acme Spacecraft

[34] Humor is the mask of wisdom.

[35] The earliest joke book in the English language, published in 1739, based originally on the jokes of hapless stage comedian Joe Miller, who had died in poverty the year previous. As *Joe Miller's Joke Book* it found its way frequently into Mickey Mouse and Donald Duck comics of the 1940s and 1950s, for which I had a deep affection.

[36] The word *comedy* derives ultimately from Greek *κῶμος*, "revel," and Greek *ᾠδή*, "song." The Ancient Greek *κωμῳδία* has much the same sense as the modern English word but was, perhaps, more exuberant than today's comedic entertainment. The word *tragedy* comes from Greek *τράγος*, "he-goat," and, again, Greek *ᾠδή*, "song," and, therefore, means "goatsong," possibly a song sung (or poem recited) in a play while clad in a goat skin, but the original meaning is unclear. By obvious extension *τράγος* could also denote "the aroma of armpits," which adds an entirely different flavor. In any event, even in ancient times the word came to denote a stage production of a serious nature. The word *drama* derives from Greek *δρᾶμα* and means, among other things, a "stage performance" (from the verb *δράω* "to do" or "to be doing." It was not restricted to productions of a serious nature, as it is in modern English. The French expression *comédie dramatique* hints that *comédie* might once have had a wider meaning in that language. An actor in French is still called "un comédien" regardless of genre.)

[37] I had tested the waters the year before in a conference paper of the same work.

Company is, thus, a ship without a rudder. It has never had a contract and, of course, it has no income. Its sole "employee" works *pro bono*. Nonetheless, I have published more journal articles under its aegis than I had for any real employer.[38] Because the Acme Spacecraft Corporation has showed up as my affiliation in my publications during the past year and a half, one engineer, thus far, has asked me for employment. Also, about this time I began offering Landis Markley *begrudging* thanks in my acknowledgments, because he always found many mistakes in my papers.

In the journal article "Stellar Aberration and Parallax: A Tutorial" [22] my humor took new directions, the footnote and the bibliographic reference.[39] The technical footnote on affine spaces may have been extreme but barely funny. Not so the linguistic footnotes in which I painstakingly presented the etymology of the word *fiducial* and the motivation from German and French for my choice of coordinate-system symbols. In the Reference section I took the opportunity to tell the reader a funny anecdote about one cited Einstein work and for another to cite a journal volume which I assured the reader he would never find, because essentially all copies had been stolen from libraries for collectors.

The apotheosis of my journal humor came certainly in the article "Attitude Analysis in Flatland: The Plane Truth" [23], Here there was humor even in the title, in which "plain truth" was replaced by "plane truth," obviously, the two-dimensional variety. Flatland, of course, was the title of the celebrated little book by E. A. Abbott [24], on the problem of three-dimensional creatures understanding a two-dimensional world and a satire on human understanding. Every section heading of my article quoted a passage from the book. Thus, the Discussion section opened with "... my Lord has shewn me the intestines of all my countrymen in the Land of Two Dimensions ...," and the acknowledgment with (appropriately) "You see ... how little your words have done." These humorous inclusions were also present in the earlier conference report at a symposium at Goddard Space Flight Center [25], at which everyone knew each other and the event was always fairly lighthearted, but the journal article went even beyond these. There were long footnotes on the etymology of "quaternion" and my newly invented "binion," touching particularly on the fact that these words had Latin stems but Greek endings and on the absence of distributive numerical adjectives in Ancient Greek, an observation central to Astronautics. Nonetheless, I tried to invent suitable Ancient Greek words for the quaternion, of clear technical importance. There were references not only to Latin and Ancient Greek but also to Classical Persian and the poet Omar Khayyam, and I was careful to point out that the Classical Persian word for "quatrain" (*rubaï*), an obvious relative of quaternion, was really of Arabic origin.[40] That was not all. The earlier conference report [25], as an example, had examined the Flatlandish equivalent of the unconstrained quaternion Kalman filter. Since I was about to publish a more complete study of unconstrained quaternion estimation [26, 27], it

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[38] This was possible only because nearly all of the articles had existed previously in typeset form, and had already been submitted or even accepted long ago. One cannot produce the research for fourteen journal articles, or even write that many for previously completed research over a period of six or seven weeks. Yet my friend John Junkins has called this "the greatest burst of creativity" he had ever seen. It would be more accurate to call it "the greatest burst of laser printing."

[39] An example of the comedic footnote appears in this section.

[40] Modern speakers of Farsi, Arabic and Hebrew say "quaternion" in their respective languages to denote the mathematical object.

was clear that a new example was needed. I chose the OLAE estimator of Mortari, Markley and Junkins [28], three of my closest friends with "attitude problems." I wanted a snappy sounding name like OLAE (pronounced *¡olé!*) for my two-dimensional version, and I wanted to poke fun at algorithm names (and at my friends), so I called my algorithm the Optimal Ingenious Vainglorious Attitude Estimator (OIVAE), whose acronym was pronounced like the Yiddish "oy vay," as in "oy vay iz mir" (= oh, woe is me!) with the quotation in both Roman and Hebrew letters in the footnote. To top things off, in the author footnote I made myself acting manager of the Planecraft Division of the Acme Spacecraft Company in addition to my more exalted position of director of research. Clearly, the editors of *The Journal of the Astronautical Sciences* were in on the joke. The paper was dedicated to John L. Junkins and included in the special issue of the JAS celebrating his sixtieth birthday. I like to think that this combination of self-parody and serious content was a better present for my friend than a more sober article. I had reached my peak, and it is now time for moderation. *μέτρον ἄριστον*, moderation is best.

This is only a small sample of the way in which humor can be a part of Astronautics publications, and not even all of my own examples. The possibilities are endless. There will be more, but never gross or hurtful humor. *Sic transit gloria mundi*.

My Proudest Accomplishment

μικροῖς πόνοις τὰ μεγάλα πῶς ἕλοι τις ἄν;[41]

Euripides (480–406 BCE)

Perhaps, I can be allowed the self-indulgence at the end of my keynote address to tell you what I think is my most significant contribution (and my proudest) to Astronautics. It is not QUEST *per se*, or the survey paper I wrote with Gene Lefferts and Landis Markley on the Kalman filter for spacecraft attitude, or my mammoth survey paper on the attitude representations. It is a small section which appeared in my first journal article in Engineering (on the TRIAD and QUEST algorithms).

The model is quite simple. If $\hat{\mathbf{W}}_k$ is the k-th observed direction (in the body frame), $\hat{\mathbf{V}}_k$ the corresponding direction in the primary (space) reference frame, and A_k the attitude at the time of the k-th measurement, then

$$\hat{\mathbf{W}}_k = A_k \hat{\mathbf{V}}_k + \Delta\hat{\mathbf{W}}_k\,, \qquad k = 1, \ldots, N$$

where the measurement noise terms $\Delta\hat{\mathbf{W}}_k$, $k = 1, \ldots, N$, are assumed to be Gaussian, zero-mean, mutually uncorrelated, and with covariance matrix

$$R_k = \sigma_k^2 \left[I_{3\times3} - (A_k\hat{\mathbf{V}}_k)(A_k\hat{\mathbf{V}}_k)^{\mathrm{T}} \right]\,, \qquad k = 1, \ldots, N$$

This is all there is to it. It is a very trivial model. Yet no one had thought to propose it before.

When I needed a measurement model to calculate the estimate-error covariance matrix of the QUEST algorithm, I wrote down the one above in a flash without having

[41] How could one by little toil achieve great things?

to think about it. It is by far my most useful result, and, I believe with all my heart, more valuable than QUEST. Starting from this measurement model one can even derive the Wahba problem as the maximum-likelihood estimator of the attitude [29], an important result. If the Wahba problem is the Kepler problem of Attitude Estimation, then this model must be the equivalent of the inverse-square force. Since the model's first application was in my QUEST work, I have come to call it the QUEST measurement model.

The result is not even exact. For example, the mean of $\Delta\hat{\mathbf{W}}_k$ cannot be exactly zero but is of order σ_k^2. However, since σ_k is generally a very small number, anywhere from 0.01 radians down to .000005 radians (or a thousand times smaller for the fine guidance sensors of the Hubble Space Telescope), so the neglect of second-order terms is justified. In addition, the model assumes that the sensor has circles of error in the tangent plane to the measured direction, instead of the more correct ellipses of error in the focal plane of the sensor. Therefore, the model is realistic only for sensors with a small field of view. The QUEST measurement model has many flaws. Yet in its own way, it is magnificent and extremely useful. It is my proudest accomplishment in Astronautics.

A Final Word

τὸν καλὸν ἀγῶνα ἠγώνισμαι.[42]

II Timothy [4:7]

Read great literature (novels, short stories, plays, poems, and even children's literature); ponder great pictorial and plastic art; watch great movies; listen attentively to great music; study foreign languages old and new and read their literature in the original as well as in translation; study Philosophy, History, religions (especially the ones that aren't your own), Psychology, Mathematics, Physics, and Biology; dine on exotic cuisines, travel; and engage in sports if you are able. Simply seek out every (legal) kind of stimulation, mental and physical, that you can. Also, live a good life, be kind and caring toward your fellow man, teach our youth, learn to laugh more, do great work, and write good papers. Above all, *know who you are*, an exhortation dear to the ancient Greeks, *γνῶθι σαυτόν* (Know thyself!), which implies that you must first *think* about who you are.[43]

I thank all of you for coming here this week, and I thank the organizers for this wonderful event. Even more important than the sure knowledge that one's professional work has been appreciated is the assurance that he has friends. The presence of so many friends at this event is more valuable to me than gold. The symposium has made me very happy, however unworthy I may feel of all the fuss and attention. In the last words of the Roman emperor Augustus (63 BCE–14 CE):

ACTA EST FABULA, PLAUDITE[44]

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[42] I have fought the good fight

[43] This exhortation, attributed to the great Greek astronomer and philosopher Thales of Miletus (624–547 BCE), was chiseled over the gate to the oracle's compound at Delphi.

[44] The story is done, applaud

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