

AAS-01-250

IN QUEST OF BETTER ATTITUDES

Plenary Lecture*

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For Paul Davenport, who showed the way

PROLOGUE

This lecture is, I hope, the first of a series of annual lectures by the most recent recipient of the AAS Dirk Brouwer Award. Given the occasion, I suppose it inappropriate that I give a talk in my usual style, which is rich in equations and mathematical derivation. Instead, I will try to give a talk rich in perspective and personal anecdotes.

Since the word *quest* appears in the title, it might be assumed that I will devote part of this talk to the QUEST algorithm, certainly my best known work. What is not known generally, however, is that the QUEST algorithm was the subject of my very first task in spacecraft attitude determination and that the work was accomplished almost entirely in my very first year in Engineering. QUEST was not, as many believe, created by a “renowned expert” on spacecraft attitude applying his considerable knowledge and experience. Rather, it was the lucky creation of a newcomer who had no training and no experience in spacecraft attitude determination or in any part of Astronautics, someone who simply stumbled along obstinately until he reached his goal. That this somewhat clumsy creation would become the most widely-used spacecraft attitude determination algorithm in the world today has surprised no one more than its creator.

It is hard from these remarks to escape the conclusion (not a happy one for me) that my Engineering career must have peaked very early, in its first year, in fact, and then for the next 23 years has been in constant decline. You might expect, therefore, that the QUEST algorithm is the last thing I would want to talk about. So, of course, the QUEST algorithm and how I came to develop it will occupy almost all of this talk. But I do not wish to spend an entire hour deriving QUEST. Instead, I wish to talk about the circumstances of QUEST’s birth, its adolescence, and its adult life. I wish also to talk about QUEST’s recent competitors. And I would like to talk about how my early work on QUEST has influenced so much of what I have done in the past two decades. If I am still known best for my first year’s work in Spacecraft Attitude Determination, it is because that work has been remarkably fruitful.

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DRAMATIS PERSONÆ

To appreciate the development of the QUEST algorithm fully, one must know something of the development of its creator at the time. My work on QUEST began only a few months after I entered the world of Engineering. This took place in May 1977, when I joined the Attitude Systems Operation of the Computer Sciences Corporation in Silver Spring, Maryland. Before then I was a theoretical nuclear physicist. I was, in fact, a pretty good nuclear physicist. Many of my journal articles in Physics are still cited today and occasionally someone even sends me a Physics Ph.D. thesis to read, the most recent less than a year ago. My personal life as a physicist was pretty interesting as well. For one thing I got to change my official country of residence six times. In Paris I led the life of a yuppie bohemian zipping around in my red convertible and surviving the dubious pleasure of having a knife held to my throat in the Paris Métro. In Germany I was forced to resign my university position by the Third Reich, not an easy accomplishment in 1973. In Israel I was nearly gunned down by the bodyguard of the then Minister of Defense, Shimon Peres. In Pittsburgh, my final stop as a physicist, I dated the estranged wife of a local drug lord. It's hard to imagine that anyone would want to abandon such a life, but I had many reasons in 1977 for wanting to make a career change. And so, at 4:30 p.m. on Friday, May 14, 1977, I bid farewell to my life as a nuclear physicist, and at 8:30 a.m. on Monday, May 17, 1977, I suddenly found myself employed as a rocket scientist. My life has not been the same since.

In the beginning, obviously, I was a very deficient rocket scientist. The only Engineering course I had taken previously was a sophomore course on Electronic Circuits, which I failed the first time and had to repeat.* Even worse than that, as a theoretical nuclear physicist I had become very proficient at Quantum Mechanics, but except for the undergraduate Physics courses that I had taught, I had had very little contact with Classical Physics. I was far less comfortable, in fact, with Rigid Body Mechanics than I was with Relativistic Quantum Field Theory, a fact that will resurface repeatedly in this talk.

My situation at CSC was not unique. When I interviewed at CSC I found that many of my old Physics friends had made the transition before me. One of these was Gerald M. Lerner, fellow nuclear physicist and my graduate-school roommate, who would receive the usual small bonus for having brought me into the company. Another was James R. Wertz, who had written his Ph.D. thesis in Cosmology and who, as an undergraduate, had been my neighbor in our dormitory at M.I.T. A third derelict physicist friend was Landis Markley, who had earned his Ph.D. in Elementary Particle Physics and had then gone to the University of Maryland as a postdoctoral fellow in the very year in which I was beginning my doctoral research there. Landis was my frequent companion at the daily Physics Department tea. For fifteen months the four of us worked together at CSC. And then, one by one, within a period of six months, all three of them left. I don't think their departure was in any way my doing, but if it had been, they have been much too kind to say so. *Spacecraft Attitude Determination and Control*,¹ Jim's masterwork and the first of many excellent books he would edit, appeared just after his permanent departure for sunny California. By this time (September 1978) QUEST had already been published in a conference proceedings² and most of the matter of this talk had already become history. But I am getting ahead of my story.

THE GOOD OLD DAYS – COMPUTING IN THE SPACE AGE

The young engineer today can hardly imagine what it was like to carry out computations in the late 1970s. In the 1950s a computer was most often a human being with a Marchant or Frieden calculator, noisy electrically driven mechanical contrivances only one step removed from

*Of the 162 students enrolled in this course only four received a grade of "F."

an abacas. What few electronic computers existed were exceedingly slow and unreliable. Only in 1960 did IBM, with its 1400 series, introduce computers which relied on transistors rather than vacuum tubes. The IBM 360 series, which debuted in 1964, was the first to use integrated circuits. Microchips were still a long way off.

The clock frequency of the IBM-360 series was somewhere between 500 kHz and 1 MHz. The top-of-the-line model, the IBM-360 Model-91, had a whopping 4 MB of RAM, which was called (magnetic) core in those days. Disc drives were the size of a home washing machine and had a capacity of only about 5 MB. Tape was the frequent medium for long-term and short-term storage. Computation on such a computer system was arduous. A trivial 200-state vibration analysis which I carried out on an IBM-360 Model-91 computer in 1980 required 3.5 MB of core, three disc drives and six tape drives. In order for me to have access to that much core it was necessary to shut down all systems except the operating system while I monopolized the computer from midnight until 6:00 a.m. The same task might be accomplished in a few minutes today (2001) on a student's unspectacular notebook computer boasting a clock frequency of 1 GHz, 128 MB of RAM, and a 15 Gb disc drive, computer power undreamed of in a mainframe only 20 years ago.

Operating systems in those days were equally lame. For its users IBM had created Operating System 360 Job Control Language, one of the more atrocious cruelties perpetrated against humankind. There was no virtual memory allocation in the IBM-360 series, that is, the computer would not automatically swap data between core and the disc drives. Hence, the movement of data from core to disc drives or tape drives or back had to be programmed explicitly by the user in OS-360 JCL. Every array in the program had to be specified. Instructions had to be sent via JCL to the system operator to mount or dismount tapes. Writing OS-360 JCL was an arcane art, as JCL was slightly more difficult to interpret than Sumerian cuneiform. And the few available computers were overworked. My CSC colleagues in Orbit Determination at the time will surely remember waiting days for their their programs to be executed. Twenty years into the Space Age computing was a highly frustrating task.

THE GOOD OLD DAYS – ATTITUDE DETERMINATION

What was attitude* determination like in those days? For all practical purposes there were only two methods: Batch Least-Squares Estimation and the TRIAD Algorithm, which at that time was known more commonly as the Algebraic Method.¹

In the TRIAD method, whose earliest publication seems to be by Harold D. Black³ but which possibly existed even earlier,⁴ one is given two unit vectors, the observation vectors, $\hat{\mathbf{W}}_1$ and $\hat{\mathbf{W}}_2$, which are two directions measured in the spacecraft body frame. These correspond to two unit vectors in the inertial reference frame, the reference vectors, denoted by $\hat{\mathbf{V}}_1$ and $\hat{\mathbf{V}}_2$. Ideally, in the absence of measurement noise, these satisfy

$$\hat{\mathbf{W}}_1 = A \hat{\mathbf{V}}_1 \quad \text{and} \quad \hat{\mathbf{W}}_2 = A \hat{\mathbf{V}}_2, \quad (1)$$

where A is the attitude matrix, a 3×3 proper orthogonal matrix,⁵ for which one wishes to solve. In general, a solution will not exist, because the observation vectors are corrupted by measurement noise. But we can always force a solution by defining first

$$\hat{\mathbf{r}}_1 = \frac{\hat{\mathbf{V}}_1}{|\hat{\mathbf{V}}_1|}, \quad \hat{\mathbf{r}}_2 = \frac{\hat{\mathbf{V}}_1 \times \hat{\mathbf{V}}_2}{|\hat{\mathbf{V}}_1 \times \hat{\mathbf{V}}_2|}, \quad \hat{\mathbf{r}}_3 = \hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2, \quad (2a)$$

$$\hat{\mathbf{s}}_1 = \frac{\hat{\mathbf{W}}_1}{|\hat{\mathbf{W}}_1|}, \quad \hat{\mathbf{s}}_2 = \frac{\hat{\mathbf{W}}_1 \times \hat{\mathbf{W}}_2}{|\hat{\mathbf{W}}_1 \times \hat{\mathbf{W}}_2|}, \quad \hat{\mathbf{s}}_3 = \hat{\mathbf{s}}_1 \times \hat{\mathbf{s}}_2, \quad (2b)$$

*By attitude, without any qualifying adjectives, I will always mean three-axis attitude.

and then setting

$$A = [\hat{s}_1 \ \hat{s}_2 \ \hat{s}_3] [\hat{r}_1 \ \hat{r}_2 \ \hat{r}_3]^T, \quad (3)$$

where the brackets denote two matrices labeled by their column vectors and the T denotes the matrix transpose. The matrix A is always proper orthogonal and satisfies the first of equations (1) exactly. If there is no measurement noise, the second of equations (1) will also be satisfied.

The TRIAD algorithm is of limited use in that: (1) it assumes that the measurements are unit vectors, and (2) it can make use of only two unit-vector measurements. The first limitation is not very damaging in practice, because most attitude sensors do furnish a direction, usually that of the Sun, one or more stars, the magnetic field, or the nadir. The second restriction is more of a problem, since it limits the accuracy of the attitude estimates.

When one doesn't have unit-vector measurements or one has more than two of them, one must resort to a least-squares algorithm, often called an optimal method. In this case, one writes the measurements as

$$\mathbf{z}_k = \mathbf{f}_k(A) + \mathbf{v}_k, \quad k = 1, \dots, n, \quad (4)$$

where \mathbf{z}_k is a measurement vector, $\mathbf{f}_k(\cdot)$ is some known vectorial function, and \mathbf{v}_k is the noise vector, assumed to have zero mean. The optimal attitude matrix is then taken to minimize the cost function

$$J(A) = \frac{1}{2} \sum_{k=1}^N [\mathbf{z}_k - \mathbf{f}_k(A)]^T W_k [\mathbf{z}_k - \mathbf{f}_k(A)], \quad (5)$$

where W_k is a weight matrix, necessarily positive definite. For vector measurements, such as we use in the TRIAD algorithm,

$$\mathbf{f}_k(A) = A \hat{\mathbf{V}}_k. \quad (6)$$

and \mathbf{z}_k is just the observed direction $\hat{\mathbf{W}}_k$. Estimation Theory⁶ tells us how to choose the weight matrices W_k as well.

We cannot optimize the cost function directly in terms of the nine elements of A , because only three of them can be independent. Hence, we write A as a function, say, of the 3-1-3 Euler angles,

$$A = R_{313}(\varphi, \vartheta, \psi) = \begin{bmatrix} c\psi c\varphi - s\psi c\vartheta s\varphi & c\psi s\varphi - s\psi c\vartheta c\varphi & s\psi s\vartheta \\ -s\psi c\varphi - c\psi c\vartheta s\varphi & -s\psi s\varphi + c\psi c\vartheta c\varphi & c\psi s\vartheta \\ s\vartheta s\varphi & -s\vartheta c\varphi & c\vartheta \end{bmatrix}. \quad (7)$$

The new cost function $J(\varphi, \vartheta, \psi) \equiv J(R_{313}(\varphi, \vartheta, \psi))$ is now minimized by an iterative procedure such as the Newton-Raphson method. From the complexity of equation (7) it is obvious that $J(\varphi, \vartheta, \psi)$ is a very ugly function and that such a minimization must be very tedious. Thus, if one could not use the TRIAD algorithm, the computation of spacecraft attitude, given the computational resources of the times, was very slow. Spacecraft Attitude Determination was not for the faint-hearted.

THE GATHERING STORM

The two methods just discussed were adequate for ground-based spacecraft attitude determination until the late 1970s. However, a trend was developing in which mission requirements were becoming more demanding both in terms of the required attitude accuracy and in terms of the required attitude computation rate. As the accuracy of the attitude sensors increases so does the amount of data processing that the sensor outputs require. When the computation frequency increases as well, the total computational burden increases still further. When a mission required that attitude be computed only once per minute and with an accuracy of only one degree per

axis, the current algorithms and computational resources were more than adequate. However, were attitude needed to be computed once per second with an accuracy of 20 arcseconds per axis—a 60-fold increase in computational frequency and a 200-fold increase in accuracy—the current computer system would be sorely burdened. And this situation was fast approaching.

ENTR'ACTE

My first undistinguished efforts at CSC weren't in attitude determination at all, but in attitude dynamics and control. When I first walked through the door at CSC in February 1977 for my interview, I knew nothing about either estimation or control and had learned only a few weeks earlier that the attitude of a spacecraft did not refer to its emotional character. Supposedly, I did know something about dynamics, because I had spent the previous half-dozen years as an assistant professor of Physics, which inspired some trust in me. That trust was exaggerated, but, as I was looking for a job, I did my best to encourage it. Fortunately, I got some lucky breaks.

I was allowed to spend additional time at CSC around my interview, so that I could try the job on for size. So for two days there I tried to understand some puzzling simulation results—not puzzling to me, since I had no idea what to expect—on the steady-state pitch rate during attitude acquisition of a spacecraft then under construction for NASA. By the second day, still without much to show, I was in my usual state of panic before a deadline and cursing myself that I had been so foolish as to expose my unsuitability for the work. Then, by a stroke of luck, using a trick from Quantum Scattering Theory, I was able to arrive at an easily calculable expression for the steady-state pitch rate, which could be computed for the entire range of control system parameters in much less time than would be required for repeated simulation of the attitude dynamics. My result agreed with the simulations but did not really explain what was going on any better than the simulations themselves. It did, however, increase confidence in the simulation results, which, I guess, was worth something. As a result, some months before I received an offer of employment, CSC added my name to a conference paper⁷ in which my expression and its derivation appeared as an appendix. My greater achievement was in fooling the company that I really knew something. At the very least I knew that I would not be completely lost in industry.

When I arrived for work at CSC in May 1977 I was assigned the task of determining whether the Magsat spacecraft could meet its attitude determination accuracy requirement. This was not really a problem in attitude determination but rather in attitude dynamics. The important question was: would the spacecraft without (redundant) pitch-rate gyros be able to maintain its angular velocity within appropriate limits (± 200 arcsec/sec) for sufficient time to carry out star identification? If one could identify stars, then one could process the star tracker data, in which case it was clear that the attitude determination accuracy requirement would be satisfied. My lack of experience was also clear. Fortunately, as a collaborator on this study I was able to work with Dave Gottlieb, a former astronomer and the creator of SKYMAP, a computerized star catalogue still an essential component of spacecraft attitude work, who really understood star trackers and who was the angel on my shoulder.

Well, this problem too had a quantum mechanical analogy, which was similar to the maximum-time problems that Physics graduate students often are forced to solve using semi-classical approaches to the Heisenberg Uncertainty Principle. So once more after some initial panic I was able to use my background in Quantum Mechanics to solve an attitude problem. Of course, when I presented my results I made no reference to Quantum Mechanics or to the Heisenberg Uncertainty Principle, so it looked as if I had come up with this (admittedly clunky) method all by myself. What we were able to show finally at the end of two months was that without the pitch-rate gyros, the control system wouldn't always maintain the pitch rate within appropriate limits for sufficient time, and the attitude determination accuracy requirement wouldn't be met.

Consequently, the Johns Hopkins University Applied Physics Laboratory, the prime contractor for Magsat, put the gyros back into the spacecraft design. In addition, Dave convinced APL that the boresights of the two star trackers should not be parallel. This seems obvious today, and it shows you how little people knew about attitude determination 25 years ago.

CSC was convinced now that I could walk on water. I, on the other hand, was unconvinced that I could even swim in it for long. I still knew very little about attitude dynamics and control* and nothing at all yet about attitude determination. At night I was working anxiously through the nearly 900 pages of Jim Wertz' book in progress,¹ of which I had been given a manuscript copy, trying to learn enough so that I wouldn't fall flat on my face too dishonorably. CSC was expecting me now (August 1977) to find a faster way to determine attitude for the Magsat mission, which would have the highest attitude computation rate of any NASA mission ever flown as well as the tightest attitude determination accuracy requirement of any spacecraft ever. I, of course, had not the faintest idea of what to do.

One factor in my favor, which I didn't realize right away, was that no one really knew very much about three-axis attitude determination. Jim's book, for example, contains only about eight pages on three-axis attitude determination methods. The emphasis until then, and most of the CSC experience, was on spinning spacecraft, for which, typically, one determined only the direction of the spin axis. As it turned out, however, Jim's book contained the germ of the faster attitude determination method we needed, but it would take me a while to discover it.

MAGSAT AND SEASAT

The Magsat spacecraft^{1,8,9} to be launched on October 30, 1979, would measure the geomagnetic field with the then unprecedented accuracy of 6γ ($= 6$ nT). To meet this requirement one needed to know the orientation of the magnetometer payload with an accuracy of 20 arcsec/axis (1σ). No spacecraft flown by NASA had ever had such a high accuracy requirement. In addition, since the purpose of the mission was to create a magnetic field map, one needed magnetic field measurements spaced very closely, which meant taking measurements very frequently, in this case at intervals of 0.25 sec. No previous spacecraft had needed to compute attitude with a frequency of 4 Hz. To carry out this task, Magsat was to be provided with an Adcole Fine Sun Sensor and two Ball Brothers CT-401 fixed-head star trackers, all of which would have accuracies higher than 20 arcsec/axis. That the required attitude accuracy could be achieved in theory was not in doubt. That it could be computed quickly enough at that accuracy was. Would it take a week to process one day's worth of data? If it did, then the anticipated six months of Magsat data would require more than 3 years to process, which would be an unacceptable expense and delay. NASA had assumed that processing six months of Magsat fine attitude data would require a full year. No one, however, really knew how long it would take.** For daily attitude mission operations, of course, one could not wait months or even days for attitude estimates, so a second system of more typical sensors had to be in place on the spacecraft to provide attitude data of the usual, more modest kind.

This situation demanded that there be three attitude determination systems for Magsat (all resident on a ground computer): (1) a fine attitude system*** (MAGFINE), which would determine attitude using data from the two star trackers and the fine Sun sensor; (2) an intermediate definitive (or coarse) attitude system (MAGINT), which would process less accurate data from a coarse Sun sensor and an infra-red horizon scanner; and (3) a near-real-time system (MAGNRT), largely a stripped-down and faster version of the coarse system but which would

*From this point on I would never work on problems of attitude dynamics and control again, except in the classroom.

**In actual fact, the Magsat spacecraft survived for eight months, so the processing time would have been even longer.

***I am grateful to Dr. Franklin G. VanLandingham (an astronomer), now at the Computer Sciences Corporation, Huntsville, Alabama, for refreshing my memory on many points relating to the Magsat fine attitude system.

also use data from a coarse vector magnetometer. The near-real-time system would provide attitude results very quickly from short segments of data received directly from Mission Control for general daily attitude mission operations. The coarse attitude system would provide attitude estimates with accuracies of only 0.5 deg/axis and at a rate of only once per minute, not accurate enough nor frequent enough for good scientific work, but they would be available fairly soon after complete spacecraft telemetry was received on the ground. The fine attitude system would provide attitude estimates of the highest possible accuracy (better than 20 arcsec/axis) and at very high frequency (four times per second) but only after a very long delay. In fact, the Magsat fine attitude determination system was not expected to begin routine data processing until six months after launch, the intervening time being spent in shaking down and fine-tuning the system once there was real data. Thus, the scientists were expecting to wait a considerable time before receiving the really good attitude results. It was the fine attitude determination system for which the new algorithm was needed.

The data processing algorithms had been largely specified for all three Magsat systems with one exception: we had no idea yet what the fine attitude determination algorithm would be. In August 1977, Dave Gottlieb and I constituted the Magsat Fine Attitude task, with Dave as task leader. Dave was busy with the software specification, particularly for star identification, in which he was one of the world's experts, and I started looking for a faster way to compute attitude.

My fears were fanned by the Seasat mission. The Seasat mission* would be mapping ocean wave heights over the entire Earth, at least the wet parts. The spacecraft was scheduled to be launched on June 28, 1978, sixteen months before Magsat. The required attitude determination accuracy was 0.2 deg/axis (3σ) and would be computed at a rate of once per second. Thus, in both data processing and in the attitude computation rate, Seasat was far less demanding than Magsat, but considerably more demanding than any mission that had preceded it. It was not certain that Seasat would have an easy time maintaining throughput.

One constraint that needed to be addressed in the development of a new attitude determination algorithm for Magsat was the great dislike of quaternions frequently expressed by Roger Werking (another physicist), who was in charge of attitude operations activities at Goddard Space Flight Center, and who was also the head of the section at NASA/GSFC responsible for the attitude determination software for Magsat.** For 'hands-on' people who didn't enjoy doing mathematics quaternions were regarded as unphysical and confusing, because they could not be visualized in the same way that Euler angles could. In addition, attitude operations at NASA/GSFC required that attitudes be trended so that attitude could be interpolated at points where there were poor or no data or so that estimation errors could be smoothed out. Quaternions, however, had an undetermined sign, and if methods were not developed so that abrupt sign changes did not occur, then trending the components of a quaternion would not give meaningful results. As you can imagine, embarrassments had been created in the past by quaternionophiles, which led to Roger's interdiction. I, of course, loved quaternions. To me they were just a representation of the rotational degrees of freedom that one encounters in the quantum mechanics of electrons! Fortunately, I also respected Roger Werking, which was a good thing, because he had much better common sense than the analysts, myself included. Nonetheless, quaternions could not be avoided.

Another possibility for attitude computation that was greatly disfavored by Roger was the Kalman Filter.¹⁰ Here Roger stood on firm ground. The Kalman filter up to 1977 had not been a spectacular performer for attitude estimation, certainly not for real spacecraft, and the

*I am grateful to fellow physicist Dr. Milton Phenneger, CSC task leader for Seasat Attitude Analysis, for much information concerning Seasat.

**I am grateful to Roger Werking, racecar driver and duck hunter extraordinaire, for confirming, with much good humor, my memories of his "iron" rule of NASA/GSFC Attitude Operations.

computational burden was very high. In any event, I knew very little about the Kalman filter at the time, so it was never really an option for me.

In October 1977, I was made the leader of the Magsat Coarse Attitude Analysis task.* This meant that just about all my time would be spent on analysis, software specification, validation, and verification for the Magsat coarse and near-real-time attitude determination systems, as well as the design, development and testing of the Magsat attitude system simulator, and directing typically a half-dozen people in these activities. In other words: *real work*. Developing a faster attitude determination algorithm now had a much lower priority, and, much to my astonishment, CSC essentially put it on the shelf. But one of my less endearing traits has always been an unwillingness to let go of anything I have started. Thus, finding a faster attitude determination algorithm became my hobby and holy grail if not my official duty and responsibility, and I pursued it tenaciously in my spare moments and in the evening. No one else was going to do it, and I really felt it had to be done, but mostly I just refused to let go of the problem.

In hindsight it seems amazing that there was no stronger effort at NASA/GSFC to make our attitude determination algorithms much faster given that everyone knew that serious trouble was just ahead. The reason for this must be sought in the facts that Attitude Determination was not yet a systematic field of study and that NASA/GSFC was prepared to slug it out with the data, no matter what it took. In addition, this was a problem for NASA/GSFC attitude operations, whose people were not generally drawn from theoretical physics or applied mathematics. There were certainly many people at NASA/GSFC who might have solved this problem, but, as in many large organizations, the lines of communication were absent. So it fell to the contractor responsible for the operations software, namely, CSC. In any event, there was no concern that mission objectives would not be met, only that the time required for data processing might be excessively long.

WAHBA, DAVENPORT, AND THE HEAO MISSION

Fortunately, other people had been working on new ways of computing spacecraft attitude. After spinning my wheels for a month and getting nowhere I began investigating CSC's other missions for NASA. The HEAO (High Energy Astronomical Observatory) Mission algorithm, briefly described in Jim Wertz' book¹, provided the necessary missing link that I needed.

This missing link had its origin in 1965. In that year Grace Wahba, a graduate student in Statistics at George Washington University had a summer job with IBM Federal Systems, which was the company supporting NASA/GSFC attitude activities before CSC won that contract. Grace was working on attitude determination and posed a problem in *Siam Review*,¹¹ to wit: How would one calculate the proper orthogonal matrix A which minimized the cost function

$$J(A) = \frac{1}{2} \sum_{k=1}^N a_k |\hat{\mathbf{W}}_k - A \hat{\mathbf{V}}_k|^2 \quad (8)$$

with $\hat{\mathbf{W}}_k$ and $\hat{\mathbf{V}}_k$ as before, and a_k , $k = 1, \dots, N$ a set of non-negative weights?*** Several authors¹²⁻¹⁵ responded to this problem, all of them offering interesting but not very practical solutions. These were fine for mathematicians but not for mission support. Numerous other solutions¹⁶⁻²⁵ were proposed before 1977, which were also of little help. After the summer of 1965, Grace finished her Ph.D. and never worked on problems of spacecraft attitude again. She is now a very distinguished professor of Statistics at the University of Wisconsin, unaware, except

*I inherited this task from Dr. Menachem Levitas, who was about to leave CSC. Menachem (another nuclear physicist) was also the brother-in-law of one of my former Physics colleagues at Tel-Aviv University. The world is small indeed.

**The notation here is that of Davenport not Wahba.

for our infrequent conversations, that her first and last publication on spacecraft attitude is the cornerstone of so much important work.

The most intriguing solution to the Wahba problem came from Paul Davenport, a mathematician working at NASA/GSFC. (Are there no engineers in this story?) Paul was the NASA/GSFC monitor for attitude analysis on the HEAO mission. He is also one of the most brilliant and innovative thinkers and tinkerers in attitude determination that I have ever known. As a manipulator of equations, I think his skills may exceed even those of Markley. Paul made the next significant step leading to a faster algorithm. What Paul observed was that if one defined a matrix B according to

$$B = \sum_{k=1}^N a_k \hat{\mathbf{W}}_k \hat{\mathbf{V}}_k^T, \tag{9}$$

which I like to call the *Davenport Attitude Profile Matrix*, and if one defined further the quantities

$$s = \text{tr } B, \quad S = B + B^T, \quad \text{and} \quad \mathbf{Z} = \begin{bmatrix} B_{23} - B_{32} \\ B_{31} - B_{13} \\ B_{12} - B_{21} \end{bmatrix} \tag{10}$$

as well as the 4×4 matrix K

$$K = \begin{bmatrix} S - sI & \mathbf{Z} \\ \mathbf{Z}^T & s \end{bmatrix}, \tag{11}$$

then \bar{q}^* , the quaternion equivalent to the attitude matrix which minimizes Wahba's cost function above, must satisfy

$$K\bar{q}^* = \lambda_{\max} \bar{q}^*, \tag{12}$$

where λ_{\max} is the largest eigenvalue of K . This is Davenport's brilliant result, which is the starting point for all modern work on the Wahba problem. Since K is a real-symmetric matrix, to find the optimal quaternion, one need only construct K and then determine the largest eigenvalue and the associated eigenvector using Householder's method. This would not do for Magsat, because Householder's method was too slow given the computer resources of the time, but for the HEAO mission it was perfectly adequate since spacecraft attitude would be calculated in this way only once per hour (using a full hour of attitude data) and otherwise interpolated using gyros.^{26, 27}

Davenport never published his *q-Method* nor his earlier *Y-Method* and *R-Method*, which also solved the Wahba problem. At the time it was developed, the *q-Method* was documented along with the *R-Method* only in a CSC report²⁸ to NASA/GSFC. The *Y-Method* appeared only in NASA reports.^{29, 30} The work had been done, in fact, only within a year of my arrival at CSC, and the CSC company report was issued just as I was being interviewed for my job there. The timing could not have been more fortunate.

QUEST IS BORN

While the HEAO algorithm wasn't the solution needed by the Magsat mission, because it was still not fast enough given the computers of the day, it was the gateway to finding a faster algorithm. As you may expect, my approach to the problem was once more that of the quantum physicist.

For our further discussion let us adopt the convention that

$$\sum_{k=1}^N a_k = 1, \tag{13}$$

which does not effect the optimization and which will simplify the later discussion. Then we can write Wahba's original cost function as

$$J(\bar{q}) = \bar{q}^T [I_{4 \times 4} - K] \bar{q} \quad (14)$$

Let us now make the following notational changes: $\bar{q} \rightarrow \Psi$, $I_{4 \times 4} - K \rightarrow H$, and $1 - \lambda_{\max} \rightarrow E$. We note also that since \bar{q} is real, its transpose is the same as the transpose of its complex conjugate, otherwise known as its Hermitian conjugate, written \bar{q}^\dagger . Likewise, since K is real-symmetric, H is necessarily Hermitian as well ($H^\dagger = H$). Noting all these facts and substitutions, we can write Davenport's result as finding the value of Ψ which minimizes $\Psi^\dagger H \Psi$, with H Hermitian, subject to the constraint that $\Psi^\dagger \Psi = 1$. This is just the variational principal of Quantum Mechanics! The optimization leads straightforwardly to $H\Psi = E\Psi$, otherwise known as the energy representation of the Schrödinger Equation, where H is the Hamiltonian. This last result is the same as equation (12) except now E is the smallest eigenvalue of H . (In Quantum Mechanics one is usually interested in finding the ground state, the state of lowest energy.) I had found my way home once more.

The mad quantum physicist rides again!

Was this a crazy approach to solving attitude problems? Not in the least. In 1977 I had about sixteen years of experience and course work in Physics and only a few months experience in Spacecraft Attitude. One usually is best equipped to solve problems in one's area of greatest competence and experience. Had I been a mechanical engineer, I might have converted the Wahba problem to a problem in vibration theory. Today I have 24 years of experience in Spacecraft Attitude Estimation and Estimation in general, while my Physics experience hasn't improved at all since 1977. I solve attitude problems differently now. In fact, I stopped looking for quantum-mechanical analogies after my first year at CSC, that is, after QUEST.

Having now transformed the optimal attitude problem into the problem of finding the ground state wave function and ground state energy of a very simple system, I began to look through my catalogue of Quantum Mechanics methods in search of a way to solve the problem. At first I tried Rayleigh-Schrödinger perturbation theory, which turned out to be a waste of time. I even developed a diagrammatic language for my perturbative expansion, essentially Feynman diagrams, which proved to be an even greater waste of time, although it elicited the admiration of Landis Markley. Then I tried various non-perturbative methods to solve for the ground-state energy.* One thing that I knew already from the experience of solving Quantum Mechanics problems was that once I had found the eigenvalue, finding the eigenvector would be easy. The relevant equation is

$$\mathbf{Y}^* = [(\lambda_{\max} + s)I_{4 \times 4} - S]^{-1} \mathbf{Z}, \quad (15)$$

where \mathbf{Y} is the Rodrigues vector⁵, also called the Gibbs vector. After \mathbf{Y} was calculated, one could obtain the quaternion from

$$\bar{q} = \frac{1}{\sqrt{1 + |\mathbf{Y}|^2}} \begin{bmatrix} \mathbf{Y} \\ 1 \end{bmatrix}. \quad (16)$$

Of course, applied mathematicians and a lot of engineers know about computing eigenvalues and eigenvectors too without the benefit of a long career in nuclear physics. But I had to find my experience where it lay, and it wasn't in Engineering yet. I have been careful up to now to

*These excursions into Quantum Mechanics are described in more detail in Ref. 31, which, unfortunately, will be inaccessible to most readers.

say that I worked in Engineering, not that I was an engineer. That would come later, officially, I suppose, in May 1982, when I received a master's degree in Electrical Engineering, but by that time I already had the beginnings of a reputation in Spacecraft Attitude Determination.

What I came to realize from my nonperturbative studies was that E must be very close to zero, or equivalently λ_{\max} must be very close to unity. In fact,

$$\lambda_{\max} = 1 - J(\bar{q}^*), \quad (17)$$

and we expect J to be very small at the optimal attitude. Now, there is a ready-made equation for λ_{\max} , which is just the characteristic equation of K . Thus, λ_{\max} must be the largest solution of

$$f(\lambda) \equiv \det[K - \lambda I_{4 \times 4}] = 0, \quad (18)$$

the characteristic equation for K , which has four roots. However, we know that λ_{\max} must be close to unity. So taking unity as a starting value, we can apply the Newton-Raphson method to equation (18). The Newton-Raphson method applied to the characteristic equation is usually not a good approach to computing an eigenvalue, but we had a very good starting value, and if the attitude were observable, this method should be all right.* If one were lazy, one could just set $\lambda_{\max} = 1$ and substitute this into equation (15) to obtain the optimal attitude with all of the accuracy one needed. However, one would give up a very great advantage that comes from knowing the value of $1 - \lambda_{\max}$, as we shall see below.

QUEST does solve the characteristic equation for λ_{\max} . In solving the characteristic equation I received some help from Landis Markley, who rewrote both the characteristic equation and equation (15) in more convenient forms using the Cayley-Hamilton theorem. I had already applied the Cayley-Hamilton theorem to a two-dimensional simplification of the Wahba problem with encouraging results and would soon have done the same in three dimensions. But Landis certainly saved me time. The Cayley-Hamilton theorem, while it resulted in computational savings, made the algorithm somewhat obscure, but this obscurity may even have helped sell the algorithm by making it seem more powerful and innovative than it really was. In any event, now that I had a very fast way to compute λ_{\max} , the new attitude computation algorithm was now essentially complete.

And what about Roger Werking and his interdiction of quaternions? Well, I had several points in my favor. First, since October I was working on the algorithm entirely on my own time, so he couldn't complain. Secondly, he was eager for a way to avoid the fast approaching possible débâcle. Roger was, in fact, quite happy that I was working on a faster batch attitude determination method, even if it used quaternions, and probably had more faith in me than I deserved. Finally, Roger is a very smart guy and knew that he could always relegate quaternions to the internal workings of the algorithm and then transform them for output into attitude matrices or Euler angles.

I would not have you believe that once I had converted the Wahba problem into a nuclear physics problem, it was suddenly smooth sailing. I had to repeat derivations three or four times by different paths before I could have confidence in them. For a while I would obtain results for the attitude by different methods which were the inverses of one another. This was resolved only when I finally came to understand vectors properly, particularly the difference between an abstract physical vector and its representation with respect to an orthonormal basis. Slowly, with little to guide me, I was teaching myself the general theory of attitude, rederiving every attitude relation I came upon in my reading.** It would take some time before I had confidence in what I was doing.

*If the attitude were only marginally observable, we would expect more than one eigenvalue of the K -matrix to be close to unity.

**These exercises became the core of the survey paper on the attitude representations (Ref. 5), that I published sixteen years later.

THE METHOD OF SEQUENTIAL ROTATIONS

There was still another hurdle to be overcome. Recall that the quaternion was related to the Rodrigues vector according to equation (16). The Rodrigues vector in turn was given by

$$\mathbf{Y} = \mathbf{X}/\gamma, \quad (16a)$$

where \mathbf{X} and γ are an intermediate vector and scalar, respectively, which are calculated in the algorithm. Hence, the quaternion is given by

$$\bar{q} = \frac{1}{\sqrt{|\mathbf{X}|^2 + \gamma^2}} \begin{bmatrix} \mathbf{X} \\ \gamma \end{bmatrix}, \quad (16b)$$

When the angle of rotation is 180 deg, γ must vanish, because $|\mathbf{Y}|$ is infinite then. This can happen only because of cancellations within the expression for γ , so there must be a loss of numerical significance when the angle of rotation is close to 180 deg. For Magsat and the IBM-360 in double precision simulations showed that the angle could be as close as 180 ± 10^{-9} deg before the loss of numerical significance became greater than 1 arcsec. At four attitude computations per second, this might happen once every 20,000 years.² A reasonable man would have stopped at this point and said that the algorithm was good enough. I did not stop. Like a true inventor, I wanted my creation to be perfect beyond any practical requirement. In particular, I wanted a general algorithm for any situation, and a different mission might have an attitude for which the angle of rotation were always close to 180 deg.

One way to avoid this problem was to separate the attitude into the sequence of a known rotation followed by a second rotation through an angle significantly smaller than 180 deg. If one chose a suitable first *known* rotation and applied it to the original reference vectors to generate a new set of reference vectors, then the attitude determination algorithm applied to the original observation vectors and these new reference vectors would automatically compute the second rotation without any important loss of numerical significance. This sounds like a lot of work to add to an algorithm that needed to be fast. However, if one chose the first rotation to be one of 180 deg about one of the coordinate axes, then the rotation amounts only to changing the sign of two of the components of the reference vectors. Equivalently, one simply changes the signs of two of the rows of Davenport's attitude profile matrix B . One then uses the algorithm to calculate the quaternion of the second part of the rotation, and *mirabile dictu* the quaternion of the desired full rotation can then be obtained simply by interchanging the components of the quaternion just calculated and changing the signs of two components.² This is the essence of the Method of Sequential Rotations. I argued (incorrectly, as it turned out) that for one of the four choices of the first rotation (no rotation or a rotation of 180 deg about one of the three coordinate axes), the angle of the second rotation had to be less than 90 deg. One tested the angle of rotation simply by putting a lower bound on acceptable values of γ .

It was now February 1978 and I had, for the moment, run out of ideas for things to do to make the algorithm better. I began writing a company report.³² In April, after having done a lot more simulation (all of it submitted on punched cards—I may have been among the last programmers in North America to use punched cards) I gave a seminar at CSC entitled “Application of the Methods of Theoretical Nuclear Physics to Optimal Attitude Estimation.” Our seminar room, which could seat sixty people, was packed to overflowing. Even the president of the CSC's System Sciences Division, which then employed over 900 analysts and programmers in NASA mission support activities, showed up. I would like to think that this enthusiasm was due entirely to CSC's deep confidence in and deep appreciation of my work. Unfortunately, I think it may have been due in reality to the fact that in the politically incorrect 1970s I advertised that my talk would be preceded by a short subject: “Girls of Tel-Aviv Beach.” To my relief, the audience for the featurette stayed for the seminar as well, even the division president (a geophysicist). It was

at this seminar that I unveiled the name of the algorithm, QUEST, for QUaternion ESTimator. QUEST was the third and last time that I tried to solve an attitude problem by analogy with Quantum Mechanics.

How fast was QUEST? Early tests showed that using QUEST was 1000 times faster than calculating the optimal quaternion from the matrix K using Householder's method and orders of magnitude faster still than applying a least-squares minimization based on the Euler angles.

SPEED IS NOT ENOUGH

I now began to add extra features to QUEST. First, I wanted a formally correct way to calculate the weights a_k in order to obtain the most accurate attitude estimate. My knowledge of Estimation Theory at the time was limited to only the vaguest notions of Minimum Variance Estimation (MVE). Therefore, I reasoned, in order to find the best choice for the a_k I must minimize the attitude covariance matrix as a function of these weights. What I needed was a simple expression for the attitude covariance matrix for the Wahba problem, something I could differentiate. A more experienced person would have known how foolhardy a task this was. I was blissfully ignorant and proceeded with a boldness that comes only from ignorance.

The starting point in deriving a simple expression for the attitude covariance matrix was obviously a simple model for the covariance matrix of the attitude sensor measurements. Now, the weighting of each vector measurement in the Wahba problem was characterized by only a single parameter, the weight a_k . Hence, I reasoned, the measurement model also should have only a single parameter. I proposed

$$\hat{\mathbf{W}}_k = A \hat{\mathbf{V}}_k + \Delta \hat{\mathbf{W}}_k, \quad k = 1, \dots, N, \quad (19)$$

where $\Delta \hat{\mathbf{W}}_k$, the measurement error, is assumed to be zero-mean and Gaussian, which couldn't be true exactly but was close to the truth, and had the covariance matrix*

$$E \left\{ \Delta \hat{\mathbf{W}}_k \Delta \hat{\mathbf{W}}_k^T \right\} = \sigma_k^2 \left[I_{3 \times 3} - (A \hat{\mathbf{V}}_k)(A \hat{\mathbf{V}}_k)^T \right]. \quad (20)$$

Thus, σ_k^2 is the variance of a component of $\hat{\mathbf{W}}_k$ along any axis perpendicular to $A \hat{\mathbf{V}}_k$. This was equivalent to assuming that the measurements had a circle of error rather than the more general (and more correct) ellipse of error. This might be a poor approximation for an infra-red horizon scanner, in which the errors don't have a very symmetrical distribution (nonetheless, it is frequently used nowadays for that sensor), but for focal plane sensors with small fields of view, like the Magsat fine attitude sensors, it should be a reasonably realistic representation of the truth. I eventually called this the QUEST Measurement Model.

Given my simple measurement model and the expression for the optimal quaternion as provided by QUEST, I was now able to calculate an analytical expression for the attitude covariance matrix as a function of the weights a_k , $k = 1, \dots, N$, and the measurement error parameters σ_k , $k = 1, \dots, N$, an expression that was not very complicated. Nonetheless, the MVE condition on the a_k turned out to be hopelessly complicated, and for some time I was very disheartened.

Fortunately, all was not lost. While there was no easy way to minimize the attitude covariance matrix, I did find, however, from my analytical covariance calculations that as a function of the weights, the cost function (optimized over attitude) would be smallest if I chose

$$a_k = \frac{\sigma_{\text{tot}}^2}{\sigma_k^2}, \quad (21)$$

*The negative term in equation (20) accounts for the fact that the length of a unit vector is perfectly known, so the variance in the measured unit vector must be zero along its direction.

where σ_{tot}^2 was a constant which must satisfy

$$\frac{1}{\sigma_{\text{tot}}^2} = \sum_{k=1}^N \frac{1}{\sigma_k^2} \quad (22)$$

in order to guarantee the unit-sum condition on the weights. When I chose these optimum values of the weights, which gave greater weight to the more accurate data, I obtained an even simpler result for the attitude error covariance matrix, namely

$$P_{\theta\theta} = \left[\sum_{k=1}^N \frac{1}{\sigma_k^2} \left(I_{3 \times 3} - (A\hat{\mathbf{V}}_k)(A\hat{\mathbf{V}}_k)^T \right) \right]^{-1}. \quad (23)$$

I haven't said yet what the attitude covariance matrix was. In every application I had seen up to this point in my attitude work—not that I could claim at that point of my career to have seen very far or very much—the covariance matrix was defined as the covariance matrix of the statistical errors in the Euler angles. This is the only form you will find in Wertz' book.¹ That has to be the worst definition imaginable, however, because the Euler angles are badly behaved, and, by a phenomenon similar to gimbal lock, a minute change in the attitude can show up as a humongous change in the Euler angles. What is more, the magnitude of the Euler-angle errors will depend on the choice not only of the body reference axes but also on the choice of the inertial reference axes, although the attitude sensor errors are meaningful only in a body-fixed frame. So instead of using the Euler angles I defined the attitude covariance matrix as the covariance matrix of the rotation vector⁵ $\Delta\theta$ of the small rotation δA carrying the true attitude into the estimated attitude.

$$A^* = \delta A(\Delta\theta) A^{\text{true}}. \quad (24)$$

This definition was free of the toxic behavior of the Euler angles and did not depend on the choice of the inertial coordinate system. As I learned a few months later, I was not the first person to come up with this definition of the attitude covariance matrix, but it has certainly appeared more frequently in my publications than anywhere else, which probably was significant for its use becoming routine.

Equation (21) was very significant. Without knowing it, I had just reinvented maximum likelihood estimation (MLE).⁶ We will return to this later.

It should be obvious that my approach to attitude problems had changed by this point. For most of a year I had been lost and confused, cautiously feeling my way, and taking much too much time to discover the obvious (such as equation (17)). Now I had begun to be in control of what I was doing and even enjoying the work. New obstacles became challenges rather than defeats. Finally, I was working entirely within the context of spacecraft attitude without the aid of the rather clumsy crutch provided in earlier months by my knowledge of Quantum Mechanics. My internal transition from nuclear physicist to astrodynamist took place, I suppose, sometime in the spring of 1978. My knowledge of Engineering and spacecraft attitude was still very limited, but from this point on I was at least working on Engineering problems from the inside. What a difference a year makes, but a very strenuous year, to be sure.

THE TASTE OF QUEST

If this simple model for the attitude covariance was not enough, QUEST also provided an easily calculable figure of merit for data checking which turned out to be a greater time saver than the speed of the attitude computations themselves. Equation (17) shows that the cost

function evaluated at the optimal attitude is just $1 - \lambda_{\max}$. Hence, λ_{\max} , whose calculation is central to the attitude computation, also tells us how well we optimized.

By now I had become skilled at calculating statistical quantities with the QUEST measurement model and was able to show that when the number of unit-vector measurements N became large, the random variable

$$\text{TASTE} \equiv \frac{1 - \lambda_{\max}}{\sigma_{\text{tot}}^2} \quad (25)$$

would have to good approximation a χ^2 distribution with $2N - 3$ degrees of freedom. In practice this was a good approximation for the statistical distribution of TASTE even for $N = 3$. So typically, TASTE would have a mean of $2N - 3$ and a variance of $2(2N - 3)$. For three vector measurements, the ideal case for Magsat, this would mean that $\text{TASTE} = 3 \pm 2.44$. If something were wrong with the data (for example, if a star were misidentified, so that its assumed direction might be wrong by about one degree* then TASTE would have a humongous value on the order of 10^5 . Glints in the star trackers might result in even greater values for TASTE. For the case $N = 2$, TASTE would no longer have anything close to a chi-squared distribution, but the test still worked.** So by examining TASTE, one can validate data very quickly.

Some background is needed to understand the value of this data checking method. An important part of NASA/GSFC attitude ground support was the removal of outliers from the data. The way this was done prior to QUEST was primitive and time-consuming. Essentially, one computed the attitude estimates for a data segment and converted them to roll, pitch and yaw. An eighth-order expansion in Tschebyscheff polynomials was then fit to each of these angles and displayed on a graphics device. An analyst would then examine every curve and eliminate by hand with a light pen any data points that were far from the fitted curve. The coefficients of the Tschebyscheff polynomial expansion would then be recalculated and the values of the fit curve would become the accepted values of the attitude estimate for the missing data, in fact for any time. This primitive smoother was a very time-consuming method of data validation. The TASTE test, which was fast and could be automated, was clearly superior.

SEASAT LAUNCH

On June 28, 1978, Seasat was launched. From the beginning there were problems. Some of these were associated with the slowness of the attitude determination algorithm. A more serious problem was that the attitude data were not properly time sequenced. Since Seasat had only infra-red horizon scanners and Sun sensors for attitude data, the attitude could not be estimated properly during orbit night. (Three-axis magnetometers were present on Seasat, but were not of sufficient accuracy to meet the accuracy requirement for attitude determination.) To remedy this Al Treder at NASA Jet Propulsion Laboratory had devised a yaw interpolation algorithm which would fill in the missing yaw attitude when needed using a dynamical model. This did not work very well—and was not expected to—but it did its job within the expected bounds. Because of the limited number of attitude sensors, QUEST would not have had any special importance for Seasat.

QUEST GOES PUBLIC

QUEST was presented to the outside world for the first time at the AIAA Guidance and Control Conference in Palo Alto, California, in August 1978,² just fifteen months after my joining

*The star density of the Magsat star catalogue was on the order of one star per square degree.

**A point of caution: In the case that one direction measurement is of very high accuracy and the others of much much lower accuracy, the TASTE test may not work well, because TASTE will be dominated by the data from the less accurate sensors. This was not the case for Magsat fine attitude determination.

CSC. The covariance analysis, the optimal prescription for the weights, and the TASTE test had all been finished too late to be included in the conference paper. A month later, the QUEST work, but not the name, had the honor of being one of the last things to be included in Jim Wertz' book,¹ although it receives only a few lines, just after the presentation of the HEAO algorithm, at the bottom of page 428. Missing from the conference report, from the brief remark in Jim's book, and from any succeeding publication was any mention of Quantum Mechanics.

THE DEATH OF SEASAT

Seasat, which had been expected to be the bellwether for Magsat in some ways, continued to experience problems. The burden of these problems with the attitude data processing fell mostly on the attitude operators, who were CSC employees working for NASA on site. These dedicated souls had to put in an excessive number of extra hours and frequently forfeited their weekends. Such sacrifices became tiring after three months and led to the Seasat Revolution. This 'bloody' event took place on October 10, 1978, when two of the Attitude Operations analysts, learning that yet another weekend would be forfeit, announced on the spot that they were quitting. They had had enough. Other resignations would certainly follow. Then came one of those happy coincidences that one encounters usually only in the movies. On the very same day, within two hours, in fact, word came that the Seasat spacecraft had experienced a massive short circuit and had expired.* The threats of resignation were quietly forgotten.

Was the Seasat mission a success? Apparently so. According to one of the experimenters, George Born at the University of Colorado, the early demise of the spacecraft was a blessing in disguise, because it freed up a great deal of operations money that could now be used for data analysis. From the standpoint of attitude operations, however, the Seasat mission was surely very unpleasant, to say the least.

THE MAGSAT QUEST CODE

My official responsibility in the Magsat mission was for the coarse and near-real-time attitude determination systems, none of which I coded myself except for the TRIAD subroutine, which I made look as much like QUEST as possible, including a newly derived attitude covariance matrix for TRIAD based, of course, on the QUEST measurement model.³³ Since I already had working software, I was asked to code the QUEST algorithm for the fine attitude determination system. However much sense this made, it was a error in judgment on the part of the Magsat Fine Attitude Task. I continued to tinker lovingly with the QUEST subroutines until the very last minute, when finally I was told gently but firmly that the QUEST code was needed for end-to-end acceptance testing in two days.

In my effort to make QUEST more efficient, I had made the QUEST code somewhat murky by adding three parameters, which were thresholds for when one would invoke the method of sequential rotations, the maximum acceptable attitude error level, and the computational accuracy one wished for λ_{\max} . To these parameters I gave the endearing names QUIBBL, FIBBL and QUACC. Computing these for a given mission has often been the bane of QUEST users.

An additional level of opacity was dictated by the limitations of the computer resources of the day.** For even twenty-arcsec sensors (those on Magsat were more accurate), the value of

*Conspiracy aficionados claim that clandestine groups in the federal government or the military, alarmed that the minute wakes of submerged submarines were visible in the Seasat data, terminated the Seasat mission with extreme prejudice to the spacecraft. Others have claimed that the spacecraft never died, but that the story of its demise was invented to help staunch the flow of sensitive data. Thus far, no one has claimed to have spotted Seasat in a K-Mart parking lot.

**Limited though they seem in retrospect, they were the absolute state of the art at the time of Magsat.

λ_{\max} , the fundamental internal quantity in all of the computations, would differ from unity by only about 10^{-8} . Since this difference was crucial to the TASTE test, it was necessary that all of QUEST's computation be carried out in double precision. However, double precision was an impossible luxury for the rest of the Magsat Fine Attitude Determination software due to time constraints. Thus, the inputs and outputs to QUEST were in single precision while the internal computations were in double precision, necessitating two parallel sets of input/output and internal parameters.

THE MAGSAT LAUNCH AND MISSION SUPPORT

The Magsat spacecraft was launched on October 30, 1979, one year after the demise of Seasat. We would now see how QUEST would behave with real data. As I have said earlier, it was anticipated that fine attitude determination would require one year of data processing for six months of data, with definitive data processing not starting until six months after launch. The coarse attitude system would provide much less accurate attitude results on a daily basis as soon as orbit tapes and sufficient telemetry data became available.

Roger Werking, cautious as ever, had insisted that there be a back-up attitude determination algorithm in case QUEST didn't work. CSC's back-up algorithm (not by me) was simple. If there were only two observation vectors, TRIAD would be used. If all three observation vectors were present, TRIAD would be used for each of the three pairs, the three attitude matrices would be converted to Euler angles, and the results averaged. This *ad hoc* method would be clumsy and slow, but in the frequently very crude and unsystematic way that attitude had been calculated up to then it would get the job done. This was the algorithm that a "real" engineer might have come up with back when I began this work. I, however, was a theoretical physicist and not an engineer, and the idea of proposing an *ad hoc* algorithm that was not derived mathematically from basic principles was totally alien to me.* Fortunately, this alternative method was never needed, nor, I think, even exercised, and the Magsat mission was able to benefit from having the extra tools provided by a covariance matrix and TASTE.

Routine coarse attitude data processing began four days after launch. During the first days after launch I would frequently check the performance of the near-real-time attitude determination system by calculating the Magsat spin-axis attitude on a Texas Instruments TI-59 programmable calculator, the latest thing at the time in personal computing. I was no more trusting than Roger.

As it turned out, the fine-tuning and shakedown of the fine attitude system required five of the anticipated six months, due mostly to problems with the star-identification routines.** When the fine attitude determination system began routine processing in the spring of 1980, the TASTE test was implemented for data validation in the Fine Attitude System. However, an analyst still did data checking with the Tschebyscheff polynomial fit technique, just to be safe. To everyone's delight, the TASTE test worked very well at eliminating outliers before they could turn up on a graphics display terminal. After two weeks and not an outlier in sight, the fits were subjected only to the most cursory inspection. It was also clear by this time that because of QUEST the Fine Attitude Determination System was operating much faster than the Coarse Attitude Determination System, something which had not been expected at all. As a result, after two weeks of Fine Attitude Determination System operation, the Coarse Attitude Determination System was shut down entirely – two years of my life down the drain – and only the Fine Attitude Determination System was exercised for the remaining three months of the mission to provide data for the scientists. The near-real-time system, however, remained in operation throughout the lifetime of the Magsat spacecraft, so not all was wasted.

*I am tempted to refer to this "evil" back-up algorithm as "the Anti-QUEST."

**"Routines" because everything was coded in FORTRAN IV in those days.

Typically, the Fine Attitude System required about four hours (clock time) to process one day of fine attitude data. This time interval was smaller by a factor of 12 than the anticipated two days for processing one day of fine attitude data. Thus, the five-month backlog of unprocessed data owing to the shakedown was quickly eliminated. It was, in fact, the TASTE test which was the real time-saver for NASA, not the lightning speed of the QUEST attitude computations, a fact that is not generally known.*

ROGER AND QUEST

Was Roger Werking, iron chancellor of Attitude Operations at NASA/GSFC, won over now to quaternions? Well, maybe just a little bit. QUEST had saved NASA/GSFC (and Roger's branch) \$300,000 possibly in operational support expenditures for Magsat alone, not to mention making certain that the project scientists would see fine definitive attitude estimates for Magsat in their lifetimes. Nonetheless, Roger's fundamental animosity towards quaternions probably never abated, although it softened slightly, and he even allowed the Magsat output data tapes to use quaternions instead of the usual Euler angles. Roger certainly developed respect and appreciation for what QUEST could do and played a key rôle in its expanded use at NASA/GSFC. He also knew what his contractors could do if left to their own devices, so he decreed that the QUEST code would never be modified from the version in the Magsat software, which I had last modified only one day before acceptance testing (see below). Those of us who have watched Roger nervously break pencils in two as he approached a major mission deadline, appreciate the wisdom of his action. And so, the QUEST code as used by NASA/GSFC and its contractors remained frozen until after Roger retired from NASA, enshrining QUIBBL, FIBBL, QUACC, and the REAL*4/REAL*8 interfaces for nearly a decade. With exceptional wisdom and restraint on my part, I never told Roger that by accident I had inserted an error into QUEST when prettying up the code just two days before acceptance testing (a "+" had been replaced by an "*"), whose correction was my final modification to QUEST the following day. The next modification to NASA's QUEST software (a change in the order of certain computations to improve numerical significance), was made by Markley in 1987. I am certain that Roger, had he remained at NASA longer, would himself have caused the NASA code to be modified, certainly to meet the needs of the changing computer environment.

THE DIFFUSION OF QUEST

The diffusion of QUEST began very soon after it has proved itself in the Magsat mission. It began, naturally, at NASA/GSFC when QUEST became part of the attitude ground support system software for the Solar Maximum Mission. QUEST soon became a standard at NASA/GSFC, often replacing the TRIAD algorithm even when the latter algorithm was more than adequate.

One reason, certainly, that QUEST was adopted so quickly by NASA/GSFC was that I had daily contact with the attitude task leaders at CSC for all the other NASA/GSFC spacecraft (more than a half-dozen in preparation at any one time back in those days). I also had more than two years between my CSC seminar and the start of Magsat fine data processing in which to publicize QUEST. It was rare during the first of those two years to be within 30 feet of me and not hear about QUEST and its growing bag of tricks. All the same, immodest though I may have been back then (and since) about QUEST, I had not the faintest notion then that it would achieve the widespread fame it has today. The best thing I could say about QUEST's importance while I was at CSC was that it was probably good enough to be published in a journal.

*Nonetheless, analysts proposing alternative algorithms to QUEST generally compare only flop counts.

QUEST appeared in the *Journal of Guidance and Control* in January 1981 (Ref. 33), my very first journal article in Engineering. For an algorithm that would become so important, I had a very hard time getting it accepted for publication, which I attribute to the usual referee and editor prejudice toward new authors in a field. I had also submitted a second article, with S. D. Oh, which was devoted to a covariance analysis of the TRIAD algorithm using the QUEST measurement model. The associate editor insisted that the two articles be combined, which meant that Oh's name would be associated with the QUEST algorithm, in which she had played no part. I protested to the journal, but I had little say in the matter.

The most significant event in QUEST's diffusion came around 1987, when the NASA Jet Propulsion Laboratory adopted QUEST for its deep space missions.* If anything conferred stature on QUEST, this was surely it. JPL had also suffered from the slings and arrows of inadequate computer resources, even worse than the situation at NASA/GSFC. Deep space missions needed to be autonomous for long time intervals. Instantaneous direct control of the spacecraft was impossible from the Earth because of the finiteness of the speed of light and the very large distances to the planets. To make matters worse, the onboard software before the late 1980s did not reside on anything like an IBM-360 mainframe (try squeezing one of those into an unmanned spacecraft!) but on an Intel 8050 chip, which was less capable than the microprocessors in many microwave ovens today.**

Thus, there were enormous disincentives at JPL against using anything but the most primitive and most reliable algorithms. The success of the Voyager, Pioneer and Mariner missions attests to the soundness of JPL's judgment. By 1987, however, the microprocessor revolution was in full swing and it became possible to use a more sophisticated algorithm like QUEST with all its special features. Thus, QUEST went to Jupiter on the Galileo mission, to Saturn on the Cassini mission, to Mars on the Explorer missions, to Venus on the Magellan mission, and, as this talk is being presented, QUEST has computed attitude for the NEAR spacecraft as it touched down on the asteroid Eros. Whatever the failings of the QUEST algorithm, it has certainly gone far.

In 1989 I discovered that the Instituto Nacional de Pesquisas Espaciais (INPE) in Brazil had carried out some interesting QUEST studies. The level of Astronautics work at INPE was very high, and engineers there had even anticipated my use of QUEST as a preprocessor in the Kalman filter by several years³⁴ (see below). There soon began a fruitful collaboration with INPE and a stream of letters and even telephone calls from INPE engineers complaining good-naturedly about all the headaches they had suffered in trying to understand QUEST.*** I was by no means insensitive to the pain and suffering which I had had caused the Brazilians. As a result, for more than a decade there has hung in the secretary's office of the Department of Control and Mechanics at INPE a wooden plaque bearing a bottle of Bayer aspirin and the inscription: *From Malcolm Shuster to his colleagues at INPE*. For a while it was customary at INPE to remove an aspirin from the bottle to alleviate QUEST-aches, so much so that it was necessary to ship refills periodically from the U.S.A.

QUEST has now gained a firm foothold throughout the Solar System. It is difficult to imagine that the enthusiasm for QUEST would have been as great had the unpromising circumstances of its birth been more widely known.

*I am grateful to Dr. Fred Hadaegh of JPL (finally, an engineer!) for providing me with information about JPL's early QUEST experience.

**The Intel 8050 chip, to no one's surprise, has found far more extensive applications in microwave ovens than in spacecraft. Magsat also incorporated an Intel 8050 chip onboard in the attitude control system.

***Giorgio Giacaglia, professor emeritus of Engineering at the University of São Paulo and first head of the Brazilian Space Agency, has even commented in a course on Astrodynamics that he knew no better hazing for new graduate students than to make them rederive and understand the QUEST algorithm!

QUEST'S GREAT SHINING MOMENT

I can take no credit for what must certainly be the greatest achievement of QUEST. In early 1982 I received a telephone call from Dr. Hermann Woltring, a research fellow then at the Free University of Amsterdam, who wished to know if I had done any further work on QUEST beyond that published the year before in the *Journal of Guidance and Control*. Dr. Woltring, who died a few years ago in an automobile accident, was a biomedical engineer who had applied QUEST to the determination of limb orientation in studies of the human gait. His goal: to design better human prostheses. How much brighter must QUEST shine than all the stars and planets if it has helped a disabled child to walk.

LIFE AFTER QUEST

What did I do after the QUEST article had been published in the *Journal of Guidance and Control*? In the very month that the article appeared I left CSC and began work on submarine-launched ballistic missile systems, never expecting to work on problems of Spacecraft Attitude Determination again.

My career in spacecraft attitude determination, however, did not end abruptly at this point. My second job in the aerospace industry was at BTS, Inc., the company founded by Andrew Jazwinski, who had written a famous book on Estimation Theory and Kalman Filtering.³⁵ Clearly, I was to continue learning more about Estimation Theory and particularly about Maximum Likelihood Estimation and System Identification. At the same time, I maintained close contact with my former colleagues at the Computer Sciences Corporation, so that continued stimulation to work on problems of attitude determination, if only as a hobby, was inescapable.

Much of my work *post QUEST* was to extend the utility of QUEST or any solution of the Wahba problem. In order to determine the real σ_k for focal-plane sensors, specifically the Magsat star trackers and fine Sun sensor, I developed a method³⁶ for inferring these error levels from QUEST computations using real data. This same paper also developed a somewhat lame method for determining spacecraft attitude sensor alignments using the QUEST measurement model. That alignment estimation work has been totally superceded by Refs. 37 and 38, which assume no specific sensor error models but use the QUEST measurement model in the examples.

About eight years after the publication of QUEST, when I had just joined the Space Department of the Johns Hopkins University Applied Physics Laboratory, I showed formally that if one started with the QUEST measurement model and applied the principles of maximum likelihood estimation, one was led directly to the Wahba problem.³⁹ Thus, the Wahba problem was no longer an *ad hoc* optimization problem but belonged to the mainstream of Estimation Theory. With this knowledge, the simple expression for the QUEST covariance matrix now fell out immediately as the inverse of the Fisher information matrix. I had known this fact in an heuristic manner for some time and had used it to motivate the Wahba problem as maximum likelihood estimation of attitude in my attitude determination courses since 1983. I am not quick to publish.

Next I showed how to make the QUEST algorithm itself into a Kalman filter and Kalman smoother and developed an approximate means for simulating the effects of process noise using fading memory.⁴⁰ This suboptimal algorithm was seriously considered for the MSX mission, but in simulations I found that it missed the accuracy requirement by a factor of 2.

Since QUEST was a maximum likelihood estimator, it could be used as a measurement preprocessor within the Kalman filter.⁴¹ In more rigorous terms, the QUEST attitude solution was a sufficient statistic for the attitude, assuming the QUEST measurement model. Thus, given a star camera which measures typically the directions of 10 stars simultaneously, rather than process these 10 star directions individually in a Kalman filter, one could compute the sensor

attitude from these 10 star measurements using QUEST and then use the QUEST attitude as an effective measurement in the filter.* It is this form of the Kalman filter that I finally selected for the MSX mission. It is in this same manner that the Jet Propulsion Laboratory implements QUEST and the Kalman filter in its deep space missions. The Brazilians, as I have mentioned above, had (unbeknownst to me) beaten me to the punch here.³⁴

In this same paper,⁴¹ using a mathematical trick from Quantum Scattering Theory (old habits die hard), I also showed that one could ignore the unit-norm constraint on vector measurements and replace the QUEST measurement model in a Kalman filter with

$$E \left\{ \Delta \hat{\mathbf{W}}_k \Delta \hat{\mathbf{W}}_k^T \right\} = \sigma_k^2 I_{3 \times 3}. \quad (25)$$

This works, because the measurement sensitivity matrix cancels any contribution from the extra term in the covariance matrix.* The advantage of such a substitution is that the measurement covariance matrix for a direction measurement is now invertible. The implementation of this purely mathematical trick has been called the *unit-vector filter* by Joseph Sedlak and Donald Chu,⁴² who demonstrated that it works quite well.

It could be said that nearly half of my publications use results from the QUEST work in some way. For the most part this is because they use the QUEST measurement model either as a version of the truth or for simulation purposes. Only a half-dozen of my publications, however, are concerned directly with the QUEST attitude computation algorithm or the Wahba problem.

The work on QUEST led to to the work on the Kalman filter, but not in the way that one might imagine. At the AIAA Guidance and Control Conference in Palo Alto, California, in August 1978 Landis Markley was presenting his work on Solar Max, and I was presenting QUEST. Sandwiched between our two talks was a talk on the Kalman filtering of attitude by Jim Murrell.⁴³ Jim's work became the starting point of the work that Landis and I would do eventually on the Kalman Filter.⁴⁴ To a large degree, our work on the Kalman filter was the completion of Murrell's work.** Itzhack Bar-Itzhack was another participant in the session, so Landis and I got to know him at this time also.

ALTERNATIVES TO QUEST

It was unavoidable that alternatives to QUEST should be proposed by other workers in the field. Landis Markley and Daniele Mortari, who have been mostly responsible for the post-QUEST solutions to the Wahba problem, have published a masterful review of this work and of QUEST.⁴⁵ It is interesting to note that all of these alternative algorithms are solutions to Davenport's transformation of the Wahba problem. Occasionally the authors of these alternative methods mimic the title of the original QUEST paper in their own titles, an homage that has always touched me deeply.

*Many commercial star trackers, in fact, now output not only the star positions but also an attitude quaternion calculated using QUEST.

*It is for this reason, in fact, that Maximum Likelihood Estimation applied to the QUEST measurement model with its non-invertible covariance matrix leads to Wahba's cost function with scalar weights rather than to a cost function with weight matrices. This is the essence of Ref. 39

**Landis and I did not really collaborate on this work but had arrived at the same methodology separately for different projects, he at the Naval Research Laboratory and I at CSC. However, we frequently shared information on the telephone and used each other's results, so our work was highly synchronized and we decided to publish together. We were not very accommodating collaborators when it came to writing. The reason there were two different derivations of the results in our paper for attitude error propagation was that both he and I each insisted on our individual derivations being presented. Gene Lefferts, the first author of the paper, had not only started Landis and me on Kalman filtering of attitude, but during the writing, which took place on Saturday mornings in Gene's basement, provided us with refreshments and kept us from killing one another.

The very first of the alternatives to QUEST was Tietze's method,⁴⁶ which relied on Gaussian elimination with pivoting and inverse iteration to implement Davenport's q -Method. The approach was very reasonable, but Tietze's claims for the speed of his method as compared with QUEST turned out to be exaggerated.⁴⁷

Two of the most interesting alternatives, the SVD and the FOAM algorithms, were proposed by Markley in 1988 and 1993, respectively. The SVD algorithm⁴⁸ uses the singular-value decomposition algorithm to extract the optimal attitude matrix. The basic idea is the following. According to the Singular Value Decomposition Theorem one can always factor the Davenport Attitude Profile Matrix as

$$B = U S V^T, \quad (26)$$

where U and V are orthogonal and S is diagonal and positive definite. If UV^T is a proper orthogonal matrix, then it is the optimal attitude that minimizes the Wahba cost function. When UV^T is improper orthogonal, the algorithm is only slightly more complicated. The details are given by Markley.⁴⁸ This is a particularly attractive algorithm, because efficient robust algorithms exist now for computing the singular value decomposition.⁴⁹ Markley and Mortari⁴⁵ point out that the SVD algorithm (and also Householder's method for computing the eigenvectors and eigenvalues of a real-symmetric matrix) are both very stable, if also comparatively slow. Thus, the SVD algorithm and Davenport's q -Method with its Householder-method solution have a very special place as solution methods for the Wahba problem.*

The FOAM algorithm⁵⁰ is extremely innovative. Markley showed that the optimal attitude matrix can be written in the form

$$A^* = \alpha B + \beta \text{adj}(B^T) + \gamma BB^T B, \quad (27)$$

where α , β and γ are simple scalar functions of B and λ_{\max} , and adj denotes the adjoint matrix. Markley's form leads to a more accurate expression for the characteristic equation for λ_{\max} and a direct expression for the optimal attitude matrix. Certain complications of QUEST, in particular the method of sequential rotations, are avoided. Quaternion output can always be obtained easily from the attitude matrix. Of all the contenders, FOAM would seem to be the most worthy substitute for QUEST,** although its mathematical motivation is somewhat more obscure and it is slower (the last point is not important). However, there are possible problems with the FOAM computation of the attitude covariance matrix (see below).

Mortari has also been very active in generating alternatives to the QUEST algorithm and has published so many solutions⁵¹⁻⁵⁵ that one might even speak of an *embarrassment* of riches—and riches there are in abundance! Mortari takes his place, certainly, among the important workers on the Wahba problem and on attitude determination in general.*** It would take me too far afield and demand too much space to discuss his work here in the detail it deserves, nor could I ever hope to equal the clarity and completeness of the recent masterful review by that worker and Markley.⁴⁵ Let me say at least that Mortari's algorithms are, in general, very fast, although speed is not at all an issue now as it was 20 years ago, and all of them require the implementation of the Method of Sequential Rotations to avoid angles of rotation of either 180 deg or 0 deg. Mortari's top-of-the-line algorithm, ESOQ2, must avoid rotation angles of 0 deg,

*Landis has stated privately that the idea for the SVD algorithm came from my treatment of spacecraft sensor alignment estimation³⁷⁻³⁸ and that for a time he was afraid that I would discover the algorithm before him, a high compliment indeed, but unfounded.

**Markley, by the way, favors a quaternion-based method and is slightly inclined towards Mortari's algorithms.

***I am careful to say *attitude determination* rather than *attitude estimation*, because Mortari (in collaboration with John Junkins and others), not content just to publish very original and powerful attitude estimation algorithms, has been designing new attitude sensors as well. One might say of Mortari: Here finally is an engineer who does a great deal of fundamental work in attitude estimation without having been a quantum physicist. However, Daniele began his career as a nuclear engineer, and so belongs to that small minority of engineers whose education and career have required them to really understand Quantum Mechanics!

which it accomplishes with great efficiency. ESOQ2, the fastest of all algorithms which solve the Wahba problem, is also 10% faster than QUEST (not a very large improvement 23 years after QUEST's first appearance). The fact that ESOQ2 must use a subterfuge in order to estimate an infinitesimal rotation is troubling to this worker, but not of real consequence.

All of these alternative algorithms have had the advantage of following QUEST—just as QUEST had the advantage of following Davenport's q -algorithm. QUEST may be beginning now to show its age. Markley and Mortari have pointed out a potential problem in QUEST. In the case of only two vector measurements with vastly differing variances, say by five orders of magnitude, the QUEST algorithm does not calculate λ_{\max} as accurately as does the FOAM algorithm. If the very accurate sensor had a standard deviation of 6 arcsec, achievable with some star cameras today, the less accurate sensor would then have a standard deviation roughly 300 times greater or about 0.5 deg. This corresponds to having an attitude system consisting of a coarse Sun sensor, infra-red horizon scanner, or three-axis magnetometer paired with a star tracker which has only one star in its field of view. Such a situation is certainly possible and has occurred, for example, for the MSX spacecraft, and was anticipated. Problems of this type were already seen in simulation studies for MSX nearly ten years ago, in which the results were even more catastrophic, because the short wordlength of the on-board microprocessor led essentially to divisions by zero when computing λ_{\max} . This is not a real problem, however. The software can recognize easily when the attitude data falls into such a case and simply set $\lambda_{\max} = 1$. The resultant attitude solution will be excellent, as Markley and Mortari have observed in their review article. By not computing $1 - \lambda_{\max}$ one forgoes the possibility of using the TASTE test, but the TASTE test doesn't work well under these conditions anyway.*

A MISTAKE IN QUEST

As soon as QUEST was published, I began receiving correspondence regarding errors in QUEST. With one exception, all of these were false alarms. For the most part, the writers had tried to apply QUEST to a problem for which it was not appropriate. One writer, however, Gregory Natanson of CSC, writing to me ten years after QUEST's publication, pointed out a real mistake.

The mistake was not a mistake in the computation, but in an apparently poorly considered statement I had made that by the method of sequential rotations, the angle of rotation could always be made less than 90 deg. What Greg showed quite beautifully was that the angle of rotation could only be made less than 120 deg with certainty. This, of course, is more than adequately less than 180 deg, the angle of rotation which the method had sought to avoid. Greg never bothered to submit his result for publication. Two years later I found more general applications for the method of sequential rotations, and he and I published our results together.⁵⁶

A FINAL WORD ON MAGSAT

In 1998, while I was teaching at the University of Florida, I received a call from Mike Purucker, a contractor at NASA/GSFC, who asked if something could be done to remove the discontinuities which occurred in the Magsat attitude estimates when the sensor configuration changed. We did not know how to estimate sensor alignments correctly in 1979, when Magsat was launched, but we could do that very well and quickly now if we were given the complete

*The problem can be eliminated also by using the form of the characteristic equation in Markley's FOAM paper (Ref. 50) rather than the QUEST form, which is also due to Markley. This would increase the number of computations in QUEST but the difference in speed would be inconsequential. Markley and Mortari⁴⁵ do not test such a modified QUEST, but their review makes it clear that such an algorithm would perform as well as any other.

Magsat attitude output files, which contained the star and Sun vectors used to compute each attitude. Alas, those files had not been preserved and only the attitude quaternions had been saved, so there was nothing to be done.

THE ACHIEVEMENTS AND FUTURE OF QUEST

While challengers hoping to unseat QUEST from its privileged position try to do so on the basis of relative computational speed, speed is now the least important of QUEST's achievements, at least today when even modest notebook computers are faster by three orders of magnitude than the main frames of two decades ago. Speed was not even the most important of QUEST's achievements twenty years ago, although it was certainly important then. It was the TASTE test, even in the beginning, which was the real mission time saver. The principal achievements of the QUEST work (thus far) are: (1) the QUEST attitude computation method itself, (2) the QUEST measurement model, (3) the demonstration that the Wahba problem is the maximum-likelihood estimation problem for this measurement model, (4) the QUEST formula for the attitude covariance matrix, (4) the TASTE test, (5) the Method of Sequential Rotations, (6) the use of QUEST as a preprocessor in the Kalman filter, and, perhaps, (7) the unit-vector filter idea. Even if the QUEST attitude computation method were to be replaced in common usage, these other results of the QUEST work would certainly remain in place, some of them even as integral components of the new method, just as Davenport's q -Method is an integral component of QUEST (and of Markley's FOAM and Mortari's ESOQx). In this larger sense, QUEST is very unlikely ever to disappear from the scene.

It is well to ask at this point: What constitutes QUEST? Even for me the answer isn't very clear anymore. For almost a decade, QUEST was simply the Magsat algorithm, frozen in the MAGFINE code (called MSAD-MAGSAT in official NASA/GSFC documents). Certainly, the core of QUEST is the computation of the attitude quaternion and λ_{\max} from Davenport's K -matrix via the Cayley-Hamilton Theorem and the characteristic polynomial as well as the Method of Sequential Rotations. I would argue forcefully that the model attitude error covariance matrix and the TASTE test should be inseparable parts of QUEST as well. QUIBBL, FIBBL, and QUACC, however, or the REAL*4/REAL*8 interfaces are certainly not an integral part of QUEST nor are all of the input- and intermediate-variable checking that takes place in the Magsat QUEST code to make sure that QUEST returns an error code for really bad data rather than crashing, a necessary precaution in flight code* And is the algorithm no longer QUEST if Markley's better expression for the characteristic polynomial is substituted for the one currently in use? At the other extreme some workers even use 'QUEST' to label any executable file for solving the Wahba problem.

Which solution to the Wahba problem is best? Again there is no clear answer. The best way to compute λ_{\max} would seem now to be the application of the Newton-Raphson method to the FOAM form of the characteristic equation. Mortari has already adopted this approach in his algorithms over his previously cherished algebraic calculation with surds, and I have been speaking for years of making such a change formally to QUEST.** However, the FOAM formula for the attitude covariance matrix (and, consequently, also for the inverse attitude covariance matrix), while very pretty, will contain divisions of zero by zero if the attitude information is deficient. The QUEST calculation of the inverse attitude covariance matrix avoids these possible division exceptions entirely (and also checks the determinant of the inverse covariance matrix explicitly before inverting) but at the cost of a greater computational burden. So, with reliability

*This is the part of the QUEST code with which I tinkered up to the last minute in the development of the MAGFINE software, trying always to anticipate one more thing that could go wrong.

**Curiously, Markley and Mortari in their survey paper⁴⁵ reproach QUEST for applying the Newton-Raphson method to the characteristic polynomial to calculate λ_{\max} —"an unreliable way to find eigenvalues, in general" (p. 363)—without voicing the identical complaint for their own works. This was certainly an editorial lapse.

as the highest good and the small differences in speed of no real importance, I recommend a hybrid of QUEST and FOAM, with the FOAM form of the characteristic polynomial replacing the earlier QUEST form (both are due to Markley).

What about substituting the FOAM computation of the attitude matrix for the QUEST computation of the quaternion but retaining the clunky, but more reliable, QUEST computation of the inverse attitude covariance matrix. The resulting algorithm, alas, would be more FOAM than QUEST. If the truth must be told, I have always felt that FOAM was a more aesthetic and, possibly, a more practical algorithm than QUEST (except as noted for the covariance matrix above), even if the derivation of that algorithm is far less transparent than the derivation of QUEST, at least to me.* But FOAM came fifteen years after QUEST and, therefore, cannot compete with QUEST's twenty-one-year record of proven reliability in actual mission support. QUEST has been executed, perhaps, more than 10^{12} times with real data in more than a hundred very different missions. No amount of simulated testing can equal that. Project managers, therefore, if they are sufficiently knowledgeable, will almost always choose QUEST over competing algorithms, and with some missions costing nearly one gigadollar (US) one can hardly blame them. All the same, I would like to see a greater accumulation of flight experience with FOAM. Perhaps, FOAM (with the QUEST attitude covariance computation) will become the algorithm of the future, and QUEST will become but a memory, but greater mission experience should come first.**

Before we contemplate further the interment of QUEST by its near relations newly come to the attitude feast, let us take note that QUEST is seldom used today (as it was at first) as a stand-alone attitude determination method. Rather, it functions more frequently nowadays as simply a front end for the attitude Kalman filter.^{41,44} Soon, no doubt, it will become an even smaller component of still more elaborate systems. In this sense, QUEST's burial has long been underway. Also, as computer processing speeds continue to increase exponentially with time, one will more likely abandon QUEST in favor of the SVD algorithm or the solution of Davenport's eigenvalue problem using the Householder method, both of which are much better behaved numerically – if enormously slower algorithms – than QUEST, FOAM or ESOQ.⁴⁵ It is frequently the tortoises who ultimately win the race, not the hares. Note finally that QUEST's fast rise in popularity twenty years ago resulted very much from its having been the clipper ship of attitude determination in the days before steam. Now we have steam, at least for ground-based processing. In the long run, this writer believes, the QUEST measurement model, with its many theoretical and practical consequences – which include even the Wahba problem – will prove of more lasting value than QUEST itself. But without QUEST this model might never have been proposed. In any event, our obsequy is premature; QUEST has a lot of life yet.

EPILOGUE

If any lessons are to be learned from my history of the QUEST algorithm, which unavoidably (and for me very happily) has also been my history, they are that there is always something new to be done, and that these new things will sometimes be done by people who know least what has been done before. Expertise and experience can even be a disadvantage, since they cause us to follow well-worn paths. If I look back on the *annus mirabilis* during which I invented QUEST, when I knew nothing about spacecraft attitude, and every step was a leap into the unknown,

*With apologies to Daniele Mortari, let me state that I think that the SVD and FOAM algorithms are more beautiful and neater than our own, but beauty and neatness are not always the highest good.

**One needs more experience, for example, of how the shorter wordlength of an on-board computer or non-nominal data affect the lack of orthogonality of the computed FOAM attitude matrix. If this lack of orthogonality is too large, then my enthusiasm for FOAM will be greatly tempered.

what I do now seems to be far less exciting,^{***} even if technically of higher quality. The lessons which I myself learned while developing QUEST were enormous. I no longer stumble through attitude determination problems, perhaps, because I managed to make almost every conceivable mistake during that first year. In some sense my career in spacecraft attitude did indeed peak early. Perhaps this is as it should be.

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^{***}Exciting, perhaps, only in retrospect, though there were moments. The first part of the QUEST work consisted mostly of two years of frustration compressed into six months. Leading the Magsat Coarse Attitude Determination task, however, which contained two years of frustration of quite another kind, really was exciting as it was happening.

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