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Special Issue on Attitude Representation
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CONTENTS

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Editorial
Attitude Representation

The attitude representations must seem to be our inalterable inheritance from the great patriarchs of the past: Euler, Rodrigues, Hamilton, and Cayley, all of whom flourished more than a sesquicentury ago. We tend to regard their work as complete and immutable. Nonetheless, the past four decades have seen an explosion of new work and the need for increased capability in representing attitude. However enduring the classical work may be, it could not anticipate the demands of the Space Age and the special requirements of automatic computation. As the papers in this issue attest, there is room for considerable extension of the classical results and even for further invention. It is for the purpose of presenting some of this new work, and summarizing the old, that this special issue has been assembled.

The first work of this issue presents a survey of what is currently known about the algebraic properties of the attitude representations. The extent of that knowledge is quite large, as evidenced by the length of the survey. The vast majority of the 163 citations in that paper are to articles published since the beginning of the space age, attesting to the viability of attitude representations as a field of research.

In the second paper, Bar-Itzhack presents us with a tutorial on the Piogram, developed some three decades ago and then largely forgotten. It turns out that the Piogram often provides the simplest means for computing the solution to attitude problems in terms of Euler angles, for which Bar-Itzhack gives us many examples.

The Euler angles have been studied by the editors of this issue, who show that they can be used to develop simple derivations of the theorems of spherical trigonometry and that the spherical trigonometrical theorems, in turn, can provide us with insights into the Euler angles. In particular, they lead to a fairly compact algorithm for combining sets of 3–1–3 Euler angles, or any symmetric sets.

The computation of the quaternion from the direction-cosine matrix requires the selection of one of four algorithms in order to maintain numerical significance. Shuster and Natanson show that this quadruple multiplicity is a facet of almost any quaternion computation and present an elegant theorem which explains this multiplicity geometrically. This theorem is the basis for the development of a general methodology of quaternion computation, which is applied to typical attitude problems.

The study of spinning spacecraft is usually accomplished using the Euler angles, with one angle corresponding to the rotation about the spin axis. The Euler angles, however, are subject to well-known singularities. Markley develops a singularity-free seven-parameter representation of the attitude which associates one variable with the rapidly changing angle.

Morton's article brings to completion a study of the Lagrangian and Hamiltonian formulation of attitude dynamics in terms of quaternions. Usually, one avoids applying variational techniques to solving problems expressed in terms of quaternions because of the problems associated with the redundant variable. Morton, however, uses this redundancy to his advantage providing simple and elegant developments for both Lagrange's equation and Hamilton's equations expressed in quaternion terms. Since these are free of the extreme nonlinearities and the singularities of the Euler angles, this work presents the preferred basis for the study of the dynamics and optimal control of attitude.

Broucke also introduces a seven-parameter representation of rigid body orientation and uses this to study the nonlinear dynamics of a rigid body under the influence of a gravitational torque. Poincaré section phase-plane studies are used to reveal the geometric structure and orbital stability over the entire phase space, establishing an appealing methodology that will surely find applications in many similar problems.

Deprit and Elipe bring to completion an approach to rotational dynamics which was initiated by Serret in 1886 with the introduction of a novel geometric description of rigid-body orientation. The Serret variables and the associated canonical momenta were subsequently studied by Andoyer (1923) and are often called the Andoyer variables. In their study of the Euler-Poinsot problem, Deprit and Elipe show that the Serret-Andoyer variables are connected to an eight-parameter canonical representation, that corresponds, in fact, to the quaternions and corresponding conjugate momenta which are the subject of Morton's article. Deprit and Elipe achieve a novel and elegant solution by means of a canonical transformation which maps the Euler-Poinsot problem into a second-order action-angle system. They present a summary of the analytical solution of the Euler-Poinsot problem in terms of Jacobian elliptic functions and elliptic integrals.

We hope that this issue will acquaint the reader with the scope of the present knowledge of attitude representations and current work.

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