PAPER AAS 89-393

VERY COARSE ATTITUDE DETERMINATION FOR THE THRUSTED VECTOR MISSION

M. D. Shuster, H. L. Fisher, and K. L. Musser

The Johns Hopkins University • Applied Physics Laboratory
Laurel, Maryland 20707-6099

AAS/AIAA Astrodynamics Specialist Conference

STOWE, VERMONT, AUGUST 7-10, 1989
AAS Publications Office, P.O. Box 28130, San Diego, CA 92128

Astrodynamics 1989
Advances in the Astronautical Sciences, Vol. 71

AAS 89-393

VERY COARSE ATTITUDE DETERMINATION FOR THE THRUSTED VECTOR MISSION

M. D. Shuster,[*] **H. L. Fisher,**[†] and **K. L. Musser**[‡]

The Johns Hopkins University, Applied Physics Laboratory,
Laurel, Maryland 20707-6099

Attitude estimation algorithms are presented for the Thrusted Vector Mission which determine attitude based on Sun sensor and very coarse albedo sensor measurements. On the basis of these measurements it has been demonstrated by comparison with more accurate gyro-based attitude that it is possible to estimate three-axis attitude with an average error per axis of 11 deg. Most of this error is about the Sun direction. Both deterministic quick-look and optimal estimates are examined.

INTRODUCTION

The Thrusted Vector mission (more commonly known by its launch vehicle, Delta-181) was the second mission to be launched by the Strategic Defense Initiative Organization. The Thrusted Vector mission was divided into two phases: a data-collection phase, which lasted approximately 12 hours, during which the experimental activities of the mission took place; and a data-retrieval phase, lasting up to several weeks, during which the experimental data stored on-board the spacecraft was telemetered to Earth. Power for the spacecraft was provided by two battery systems. One of these, whose lifetime was designed to be not much longer than the experiment phase, provided power for a very accurate set of three-axis gyros. During the data-retrieval phase, it was expected that passive stabilization of the spacecraft would maintain the down-link antenna Earth-pointing with sufficient accuracy to allow uninterrupted communication during passes over the mission ground stations. Therefore, no provision was made for attitude determination during the data-retrieval phase following

---

[*] Senior Professional Staff, Guidance and Control Group, Space Department.

[†] Associate Engineer, Guidance and Control Analysis Section, Space Department.

[‡] Associate Engineer, Guidance and Control Analysis Section, Space Department. Currently, graduate student, Department of Electrical Engineering, University of Maryland, College Park, Maryland.

the loss of gyro attitude. Shortly before launch, however, it was deemed advantageous to have some means of determining attitude during the data-retrieval phase in order to verify antenna pointing. In this way one could choose the optimal antenna or even avoid downlink during passes when the spacecraft attitude might be far from nominal and the transmission would be degraded. It is to the methods developed for attitude determination during the data-retrieval phase that this paper is devoted.

In addition to the gyro assembly, the Thrusted Vector spacecraft possessed two other sensors sensitive to the attitude. One of these was a two-axis Sun sensor with an accuracy of approximately one degree per axis, which was included to provide some attitude redundancy as a check on the gross operation of the gyro assembly. The other sensor was not intended as an attitude sensor at all but to measure Earth albedo as part of the mission experiments. It was on the basis of these two sensors that attitude would have to be determined during the data-retrieval phase.

The albedo sensor provides a very crude measurement of the direction of the Earth in the spacecraft reference frame and it is certainly possible to estimate attitude based on simultaneous measurement of the Sun and Earth directions. The important question, however, is whether attitude could be determined with useful accuracy given the crudity of the albedo sensor as an attitude sensing device. It turns out, fortunately, that the question can be answered affirmatively.

In the second section of this report we present measurement models for the attitude sensors. Following this, two attitude determination algorithms are presented: a "quick-look" attitude estimation algorithm, which constructs a rough estimate of the attitude based on lumped vectors representing the observed and predicted average directions of the Earth luminance; and an optimal algorithm which attempts to improve the "quick-look" attitude by fitting individual spatial components of the Earth albedo. Complete covariance analyses are carried out in each case. Finally, an absolute evaluation of the performance of these two algorithms is presented using actual sensor data collected during the experiment phase when the much more accurate gyro attitudes were available for comparison.

SENSOR MODELS

Two-Axis Sun Sensor

Two-axis Sun sensors have been flown on nearly every mission and their operation as attitude sensors has been extensively documented [1]. A simple model of the Sun sensor errors which has proved effective in many analyses [2] is

$$\hat{\mathbf{S}}_s = A\,\hat{\mathbf{S}}_r + \Delta\hat{\mathbf{S}}_s \quad ,$$

where $\hat{\mathbf{S}}_s$ is the observed Sun direction in the spacecraft frame and $\hat{\mathbf{S}}_r$ is the corresponding vector in the primary reference frame. The measurement error, $\Delta\hat{\mathbf{S}}_s$, is assumed to be Gaussian with zero mean and covariance matrix

$$E\{\Delta\hat{\mathbf{S}}_s\,\Delta\hat{\mathbf{S}}_s^T\} = \sigma_S^2\,[I_{3\times 3} - (A\,\hat{\mathbf{S}}_r)(A\,\hat{\mathbf{S}}_r)^T] \quad .$$

The Albedo Sensor

The albedo sensor consists of 24 individual albedo detectors of approximately equal area, which together cover essentially the entire unit sphere. Geometrically, each detector is triangular. For purposes of the present analysis, however, it will be sufficient to model each detector as if the field of view were circular with a solid angle of $4\pi/24 = \pi/6$. Thus, the effective angular diameter of each detector in radians is nearly one radian. To this must be added the fact that the angular diameter of the Earth as seen from the spacecraft altitude of about 300 km, is approximately 2.5 radians. The albedo sensor is thus a very crude attitude sensing device. In addition, it is usually not possible to model the albedo of the Earth to better than 50%.

ATTITUDE ESTIMATION METHODS

Quick-Look Attitude

The quick-look algorithm uses the TRIAD algorithm [2], which computes a three-axis attitude from unit vector input. If the two observed unit vectors are $\hat{\mathbf{W}}_1$ and $\hat{\mathbf{W}}_2$ and the corresponding reference unit vectors are $\hat{\mathbf{V}}_1$ and $\hat{\mathbf{V}}_2$, then the TRIAD algorithm begins by computing the unit vectors

$$\hat{\mathbf{s}}_1 \equiv \hat{\mathbf{W}}_1 \quad , \quad \hat{\mathbf{s}}_2 \equiv \frac{\hat{\mathbf{W}}_1 \times \hat{\mathbf{W}}_2}{|\hat{\mathbf{W}}_1 \times \hat{\mathbf{W}}_2|} \quad , \quad \hat{\mathbf{s}}_3 \equiv \hat{\mathbf{s}}_1 \times \hat{\mathbf{s}}_2 \quad ,$$

$$\hat{\mathbf{r}}_1 \equiv \hat{\mathbf{V}}_1 \quad , \quad \hat{\mathbf{r}}_2 \equiv \frac{\hat{\mathbf{V}}_1 \times \hat{\mathbf{V}}_2}{|\hat{\mathbf{V}}_1 \times \hat{\mathbf{V}}_2|} \quad , \quad \hat{\mathbf{r}}_3 \equiv \hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2 \quad ,$$

and the attitude matrix is given by

$$A \equiv [\, \hat{\mathbf{s}}_1 \,\vdots\, \hat{\mathbf{s}}_2 \,\vdots\, \hat{\mathbf{s}}_3 \,][\, \hat{\mathbf{r}}_1 \,\vdots\, \hat{\mathbf{r}}_2 \,\vdots\, \hat{\mathbf{r}}_3 \,]^T \quad .$$

The covariance matrix of the attitude estimate, defined as the covariance matrix of the incremental angles which parameterize the orthogonal transformation from the true attitude to the estimated attitude [2], is then given by $P_{\theta\theta}$ with

$$P_{\theta\theta}^{-1} = \frac{1}{\sigma_1^2}[I_{3\times 3} - \hat{\mathbf{s}}_1 \hat{\mathbf{s}}_1^T] + \frac{1}{\sigma_2^2}\hat{\mathbf{s}}_4 \hat{\mathbf{s}}_4^T \quad ,$$

with $\hat{\mathbf{s}}_4 \equiv \hat{\mathbf{W}}_2 \times \hat{\mathbf{s}}_2$. The covariance matrix will be smaller if $\hat{\mathbf{W}}_1$ is chosen to be the more accurately known of the two measured unit vectors. Thus, $\hat{\mathbf{W}}_1$ in this case is chosen to be the Sun unit vector.

The quick-look method consists of finding a quick method of computing a $\hat{\mathbf{W}}_2$. To do this we define the measured luminance direction $\hat{\mathbf{L}}_s$ to be

$$\hat{\mathbf{L}}_s \equiv \text{unit}\left(\sum_{i=1}^{24} N_i \mathbf{u}_i\right) \quad ,$$

where N_i is the intensity of the albedo radiance in detector i of the albedo sensor, $\mathbf{u}_i$ is the a characteristic outward normal unit vector of that detector, and unit(·) is the operation which generates a unit vector from a vector with arbitrary but non-vanishing normalization. In all the sums over individual albedo detector elements, the elements observing the Sun are not included. The corresponding reference vector is $\hat{\mathbf{L}}_r$, with

$$\hat{\mathbf{L}}_r \equiv \mathrm{unit}\left(\int \Phi(\hat{\mathbf{n}})\, \hat{\mathbf{n}}\, d\Omega \right) \quad ,$$

where $\Phi(\hat{\mathbf{n}})$ is the radiance of the Earth albedo in the direction $\hat{\mathbf{n}}$, computed on the basis of Sun and spacecraft ephemeris and the reflectivity of the Earth.

We will assume that the luminance vector has an error representation similar to that of the Sun vector, namely

$$\hat{\mathbf{L}}_s = A\, \hat{\mathbf{L}}_r + \Delta\hat{\mathbf{L}}_s \quad ,$$

where the measurement error, $\Delta\hat{\mathbf{L}}_s$, is also assumed to be Gaussian with zero mean and covariance matrix

$$E\{\Delta\hat{\mathbf{L}}_s\, \Delta\hat{\mathbf{L}}_s^T\} = \sigma_L^2\, [I_{3\times3} - (A\, \hat{\mathbf{L}}_r)\, (A\, \hat{\mathbf{L}}_r)^T] \quad .$$

Optimal Attitude Estimation

Since the albedo sensor is so much less accurate than the Sun sensor, it makes sense to write the attitude matrix as

$$A = R\, A_{\text{quick-look}}$$

and optimize R, which was limited to rotations about the Sun vector. (Note that $\hat{\mathbf{S}}_s = A_{\text{quick-look}}\, \hat{\mathbf{S}}_r$ exactly, so this choice of R is the one which doesn't influence the agreement in the Sun direction.) Thus, the algorithm is sub-optimal. However, given a difference of an order of magnitude in the accuracy of the two sensors, little accuracy will be lost by this procedure. Thus,

$$R = R(\hat{\mathbf{S}}_s, \phi) \quad ,$$

and ϕ is the quantity over which we optimize. To carry out the optimization we consider the cost function

$$J(\phi) = \frac{1}{2} \sum_{i=1}^{24} \left[N_i^{\text{computed}}(\phi) - N_i^{\text{observed}} \right]^2 \quad ,$$

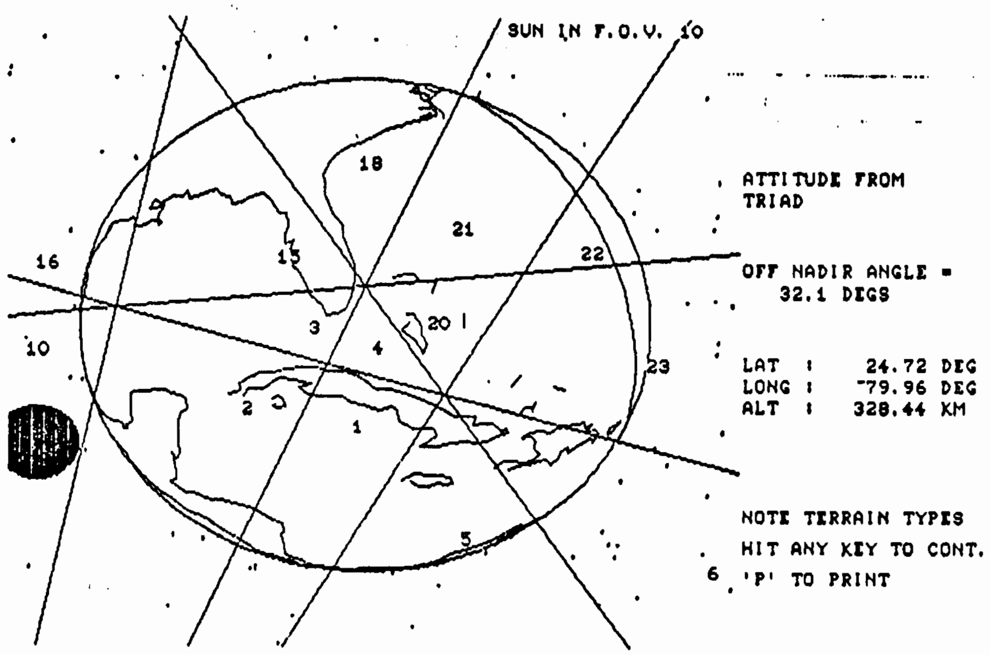
where the computed Earth albedo components are normalized so that

$$\sum_{i=1}^{24} N_i^{\text{computed}}(\phi) = \sum_{i=1}^{24} N_i^{\text{observed}} \quad .$$

The optimization was carried out by computing the cost function at five points equally spaced about $\phi = 0$, fitting a parabola to these points, and then finding the location of the minimum of the fitted parabola.

GRAPHICS INTERFACE

Figure 1 shows the graphical output that was obtained from the quick-look attitude determination. The attitude was calculated from the albedo and Sun sensor data and this was used to calculate the projection of the albedo detector boundaries on the visible Earth. The numbers in the figure label the albedo faces. The outlines of the southeastern United States, the Yucatan, and Cuba are visible. The Sun appears as a large black dot in the lower left. This does not indicate the Sun position but only the direction at which the plane determined by the spacecraft, the geocenter, and the heliocenter intersects the field of view. The terminator is visible in the right of the diagram. On the graphics screen the lit and unlit Earth were distinguished by different colors.



```
ATTITUDE FROM
TRIAD

OFF NADIR ANGLE =
   32.1 DEGS

LAT  :    24.72 DEG
LONG :   -79.96 DEG
ALT  :   328.44 KM

NOTE TERRAIN TYPES
HIT ANY KEY TO CONT.
'P' TO PRINT
```

Fig. 1 Quick-Look Attitude Graphics

Figure 2 shows the cost function for the observed and simulated albedo profiles. The fitted curve is a parabola. The minimum of the parabola gives the optimal angle of the angular adjustment to the quick-look attitude. In this case the offset is only -1.5 deg. In cases where the five points are far from being fit by a parabola, one might choose not to compute an optimal attitude by this method.

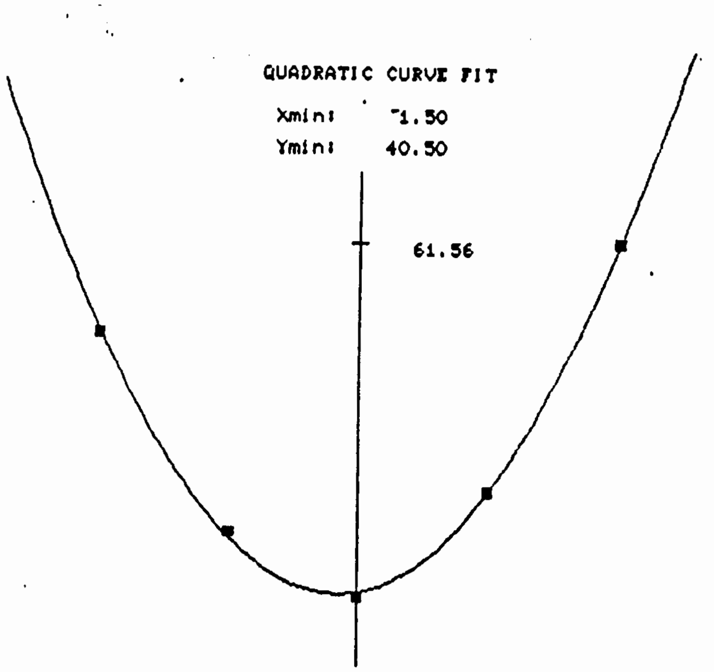


Fig. 2 Correction to Quick-Look Attitude

ERROR ANALYSIS

If we assume a uniform probability distribution for the position of the terminator or horizon in a partially illuminated albedo detector, then the sensor error standard deviations may be taken to be

$$\sigma_S = 1 \text{ deg} \quad , \quad \sigma_L = 7 \text{ deg} \quad .$$

On this basis, predictions for the quick-look attitude accuracy can be made based on the expression for the attitude error covariance given above. It should be pointed out that for the Sun directly overhead, the attitude will not be well determined, since the Sun vector and luminance vectors will be parallel. Likewise, for the Sun very near the horizon, the attitude will also be poorly determined because albedo data will be poor. If we assume for the sake of example that the Sun vector is along the x-axis and the luminance vector is in the xy-plane at an angle of 45 deg to the Sun vector, then the covariance matrix of the attitude (in rad^2) in this case using the assumed sensor variances is

$$P_{\theta\theta} = \begin{bmatrix} .03 & .0003 & 0 \\ .0003 & .0003 & 0 \\ 0 & 0 & .0003 \end{bmatrix}$$

so that two axes are determined with accuracies of about 1 deg and the remaining axis with an accuracy of about 10 deg. The average accuracy per axis (root-mean-square) for the quick-look method in this case is then about 6. deg.

We have computed a sampled covariance of the attitude errors from the quick-look method using the much more accurate gyro attitude, that was available during the experiment phase of the mission, as an absolute attitude reference. Based on sixteen samples, the estimated average covariance (averaged over different data geometries) was

$$P_{\theta\theta}^{\text{sampled}} = \begin{bmatrix} .062 & .028 & -.045 \\ .028 & .066 & -.040 \\ -.045 & -.040 & .062 \end{bmatrix}$$

corresponding to an average accuracy per axis of 14. deg. For 65 per cent of the data, however, the sampled covariance corresponded to a standard deviation per axis of 4.6 deg. The errors are, therefore, non-Gaussian. Examination of the data has shown, unfortunately, that these outlying cases could not have been identified and discarded using cosine checks of the angle between the Sun and luminance vectors. Most likely, these outliers are due to Sun glints from the spacecraft appendages and could be eliminated by suitable modeling. The accuracy needs of the mission for antenna pointing did not require that such modeling be carried out, however.

The analysis of the optimal method is more complicated but yields essentially similar results. The result for the sampled covariance is somewhat improved showing a standard deviation per axis of 11. deg.

DISCUSSIONS AND CONCLUSIONS

Simple, practical algorithms have been developed for estimating spacecraft attitude from sensors of extremely modest accuracy. Although these attitudes are less accurate certainly than those typical of even low accuracy missions, they are adequate for gross attitude estimation as might be necessary to monitor the health of the spacecraft or, as in the present instance, to verify antenna pointing. The general agreement of our error analyses with experience with actual flight data is good.

ACKNOWLEDGEMENTS

This work was presented under contract by the Strategic Defense Initiative Organization. The encouragement and support of Major Andrew J. Green, Thrusted Vector Program Manager, is gratefully acknowledged.

REFERENCES

1. Chen, L. C., and Lerner, G. M., "Sun Sensor Models," in J. R. Wertz (ed.), *Spacecraft Attitude Determination and Control,* D. Reidel Publishing Company, Dordrecht, the Netherlands, 1978.

2. Shuster, M. D., and Oh, S. D., "Three-Axis Attitude Determination from Vector Observations," *Journal of Guidance, Control and Dynamics,* Vol. 4, No. 1, Jan.–Feb. 1981 pp. 70–77.