Magnetometer Bias Determination and Spin-Axis Attitude Estimation for the AMPTE Mission

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Introduction

THIS Note describes methods for determining spin-axis attitude (i.e., the direction in space of the spacecraft spin axis) and magnetometer biases which have been investigated for ground support of the Active Magnetospheric Particle Tracer Explorer (AMPTE) mission.

The AMPTE mission will consist of two spacecraft.[1] The first is the Ion Release Module (IRM), provided by the Federal Republic of Germany, which will be placed in a highly elliptical orbit with apogee at approximately 19 Earth radii in order to release lithium tracer ions outside the magnetosphere. This spacecraft will be spin stabilized at a rate of 30 rpm. The second spacecraft is the Charge Composition Explorer (CEE), which will detect the tracer ions inside the magnetosphere at altitudes of from 300 km to 7.5 Earth radii. The CCE will be spin stabilized at 10 rpm.

Estimation of spin-axis attitude for both AMPTE spacecraft will be based on the measurements of the geomagnetic field and the projection of the Sun line on the spacecraft spin-axis, which we take nominally to be the symmetry axis Y_A of the spacecraft bus.

For the purpose of this study, the attitude sensors are assumed to consist of a three-axis magnetometer and a Sun sensor which measures the angle between the Sun line and Y_A. For simplicity it is assumed likewise that one axis of the magnetometer is along Y_A. The other two axes of the magnetometer define $\hat{X}_A$ and $\hat{Z}_A$.

The measured quantities are taken to be

M = magnetic field vector in body coordinates
$\cos\beta = \hat{S}\cdot\hat{Y}_A$, where $\hat{S}$ is the unit vector directed from the spacecraft to the Sun (β is the "Sun angle").

Attitude determination activities fall into two areas: determination of spin-axis atttitude and determination of the magnetometer biases.

Because the apogee for these two spaceraft is so great, accurate geomagnetic field data for attitude estimation are available only for the segment of the orbit near perigee. This is due to the poor accuracy of the magnetic-field model at such high altitudes, which results from both the small magnitude of the geomagnetic field as well as fluctuations in the field caused by extraterrestrial phenomena. However, because of the large spacecraft angular momenta, it can be assumed for both spacecraft that the spin-axis attitude at apogee will not differ markedly from that at perigee of the same orbit.

Algorithms for spin-axis attitude and magnetometer bias determination are now being investigated. These are 1) estimation of three-axis magnetometer bias and 2) estimation of spin-axis attitude from measurements of the Sun and geomagnetic field angle. Each of these algorithms is a batch estimator utilizing a long segment of magnetometer and Sun data. The algorithms are developed in succeeding sections and then tested using simulated AMPTE data.

Magnetometer Bias Determination

Generally, the magnetometer biases must be determined very soon after injection of the spacecraft into its orbit and before attitude information becomes available. Therefore, a bias determination procedure must be developed which is independent of the attitude. The quantities available for the estimation procedure are:

H_i = the model magnetic field in the geocentric inertial (GCI) cordinate system at time i
M_i = magnetometer reading at time i
B = the magnetometer bias vector

For the ith point, the field-magnitude error $\delta_i(B)$ is defined by

$$\delta_i(B) = |H_i|^2 - |M_i - B|^2 \qquad (1)$$

In the absence of measurement and modeling errors a value of the magnetometer bias vector can be found for which all field-magnitude errors vanish. Otherwise, the optimal value of B is that which minimizes the loss function

$$L(B) = \frac{1}{2}\sum_{i=1}^{N} \omega_i |\delta_i(B)|^2 \qquad (2)$$

where ω_i is the weight associated with the ith data point. The weights are assumed to be normalized to have unit sum

$$\sum_{i=1}^{N} \omega_i = 1 \qquad (3)$$

At the optimal estimate, $\hat{B}$, of the magnetometer bias vector

$$\left.\frac{\partial L}{\partial B}\right|_{\hat{B}} = -2\sum_{i=1}^{N} \omega_i \delta_i(\hat{B})(\hat{B} - M_i) = 0 \qquad (4)$$

Equation (4) may be recast to read

$$G\hat{B} = b + F(\hat{B}) \qquad (5)$$

where

$$G = (\langle |H|^2 \rangle - \langle |M|^2 \geq) I - 2\langle M\, M^T \rangle \qquad (6a)$$

$$b = \langle (|H|^2 - |M|^2) M \rangle \qquad (6b)$$

$$F(B) = |B|^2 \langle B - M \rangle - 2B\cdot\langle M \rangle B \qquad (6c)$$

The bracket denotes the weighted average

$$\langle A \rangle = \sum_{i=1}^{N} \omega_i A_i \qquad (7)$$

The superscript T denotes the matrix transpose, and the symbol I denotes the 3×3 identity matrix.

Equation (5) can be solved iteratively to obtain the best value for the bias vector according to

$$\hat{B}_0 = 0 \qquad (8a)$$

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$$\hat{B}_{k+1} = G^{-1}\,[b + F(\hat{B}_k)] \tag{8b}$$

where $\hat{B}_k$ denotes the *k*th estimate of B. The iteration is terminated when

$$|\hat{B}_k - \hat{B}_{k-1}| / |\hat{B}_{k-1}| < \epsilon \tag{9}$$

where ϵ is some arbitrarily small value determined by the accuracy requirements of the mission.

For $|B| \ll |H|$, this fixed-point iterative method converges. As a rule, the convergence is slowest for the component of B along the spin axis of the spacecraft (because G generally has its smallest eigenvalue along that direction). For $|B| \gg |H|$, the algorithm will, in general, not converge.

In simulations[2,3] the algorithm was found to converge more slowly than the method currently in use in support of NASA near-Earth missions,[4] which solves Eq. (4) iteratively by the Newton-Raphson method. However, while convergence is slower with the present algorithm, it occurred in all trials while the Newton-Raphson method was not always able to converge to a solution for the case of the highly eccentric AMPTE orbit, where usable magnetometer data are available for only a small fraction of the orbit near perigee.

Spin-Axis Attitude Determination

Once the magnetometer biases have been chosen properly, data from the Sun sensor and the magnetometers may be used to determine the spin-axis attitude. It is assumed that the spin axis is constant over the data interval examined.

The spin axis is denoted by a. The data are

β_i = measured Sun angle at time i $\quad i = 1,\ldots,N_S$

M_i = measured magnetic field vector at time i, (corrected for any magnetometer bias) $\quad i = 1,\ldots,N_M$

$\hat{S}_i$ = (true) Sun vector in GCI at time i, measured from the spacecraft to the Sun $\quad i = 1,\ldots,N_S$

H_i = (true) geomagnetic field in GCI at time i, $\quad i = 1,\ldots,N_M$

There is no requirement of simultaneous Sun-sensor and magnetometer data.

The spin-axis (attitude) vector, a, is subject to the constraint

$$a \cdot a = 1 \tag{10}$$

and, therefore, the spin-axis vector is chosen to minimize the loss function

$$L(a) = \frac{1}{2} \sum_{i=1}^{N_S} \omega_S(i)\, |a \cdot \hat{S}_i - \cos\beta_i|^2 + \frac{1}{2} \sum_{i=1}^{N_M} \omega_M(i)\, |a \cdot \hat{H}_i - \cos\eta_i|^2 - \frac{1}{2}\lambda a \cdot a \tag{11}$$

where

λ = Lagrange multiplier chosen to satisfy the constraint equation

$\omega_S(i)$ = weight assigned to the *i*th Sun vector measurement

$\omega_M(i)$ = weight assigned to the *i*th magnetic field measurement

The quantity η is the angle between the geomagnetic field and the spacecraft spin axis given by

$$\eta = \cos^{-1}(M_y / |M|) \tag{12}$$

and the weights are normalized to have unit sum

$$\sum_{i=1}^{N_S} \omega_S(i) + \sum_{i=1}^{N_M} \omega_M(i) = 1 \tag{13}$$

The optimal estimate of the spin-axis vector, $\hat{a}$, is a solution of

$$\left.\frac{\partial L}{\partial a}\right|_{\hat{a}} = \sum_{i=1}^{N_S} \omega_S(i)\,(\hat{a} \cdot \hat{S}_i - \cos\beta_i)\hat{S}_i + \sum_{i=1}^{N_M} \omega_M(i)\,(\hat{a} \cdot \hat{H}_i - \cos\eta_i)\hat{H}_i - \lambda\hat{a} = 0 \tag{14}$$

The solution to Eq. (14) may be written as

$$\hat{a} = (A - \lambda I)^{-1} c \tag{15}$$

where

$$A = \langle \hat{S}\,\hat{S}^T \rangle_S + \langle \hat{H}\,\hat{H}^T \rangle_M \tag{16a}$$

$$c = \langle \cos\beta \hat{S} \rangle_S + \langle \cos\eta \hat{H} \rangle_M \tag{16b}$$

and the brackets denote weighted averages over the magnetometer or Sun data. That is,

$$\langle C \rangle_K \equiv \sum_{i=1}^{N_K} \omega_K(i)\, C_i \tag{17}$$

where K denotes either M or S.

The value of the Lagrange multiplier will be given by the root of

$$f(\lambda) \equiv c^T (A - \lambda I)^{-2} c - 1 \tag{18}$$

for which the loss function is smallest. As a rule, this will be the root of Eq. (18) which is smallest in magnitude.

The solution for $\hat{a}$ may be computed iteratively as

$$\hat{a}_k = (A - \lambda_k I)^{-1} c \tag{19}$$

where the sequence λ_k is given by

$$\lambda_0 = 0 \tag{20a}$$

$$\lambda_{k+1} = \lambda_k + (1 - \hat{a}_k \cdot \hat{a}_k) / 2\hat{a}_k^T (A - \lambda_k I)^{-1} \hat{a}_k \tag{20b}$$

and the iteration is terminated when $|\hat{a}_k - \hat{a}_{k-1}|$ becomes less than some preassigned value. The estimate $\hat{a}_k$ will be a unit vector only in the limit $k \to \infty$ and, therefore, should be renormalized when the sequence is truncated. In general, λ is expected to be quite small since a is already overdetermined without the normalization constraint. If σ is the typical standard deviation of β or η, then it may be expected naively that

$$|\lambda| \approx \sigma^2 / (N_S + N_M) \tag{21}$$

The method currently in use[5] parameterizes the spin-axis vector in terms of spherical angles (θ,ϕ) and minimizes the loss function of Eq. (11) without the constraint term. Convergence is fast once the trial estimate is within a small neighborhood of the solution. However, the method currently in use does not provide a good starting value. Such a value can be inferred from the zero-th order estimate given by Eq. (19) above[6] (appropriately normalized) through that starting value is not apparent without recourse to the above derivation.

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