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In-Flight Estimation of Spacecraft Attitude Sensor Accuracies and Alignments

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A simple estimator is developed for determining in flight the accuracies of vector attitude sensors. The estimator is unbiased and independent of the configuration of the sensors. In addition, the estimator requires neither an a priori estimate of sensor accuracies nor an estimate of the spacecraft attitude. A covariance analysis of the estimator is given. Data from the Magsat mission is analyzed as an example. A simple algorithm for estimating attitude sensor misalignments, which is independent of any knowledge of the spacecraft attitude, is also presented. This misalignment estimator is applied to in-flight data from the Solar Maximum Mission.

I. Introduction

An important part of post-launch analysis for near-Earth spacecraft is the study of the performance of the attitude sensors. Spacecraft attitude sensors generally require extensive in-flight recalibration in order to achieve the desired accuracy. A re-estimate of the variance of the attitude sensor errors may be necessary not only to determine relative weights in a batch estimator or Kalman filter gains but also for evaluating sensor performance. This paper develops an estimator for the sensor variances under the assumption that the errors are unbiased. The estimator is simple and robust and is insensitive to many of the parameters needed to compute the attitude. For a vector sensor the biases can always be parameterized as misalignments. An algorithm for estimating these misalignments in an attitude independent fashion is also presented.

The attitude-sensor-variance estimator is based on a model for vector-sensor errors developed earlier[1] and applied to the covariance analysis of the TRIAD and QUEST attitude estimators. In that work the sensors are assumed to have small fields of view so that to good approximation the errors in the observed directions are distributed uniformly about the line of sight without geometrical distortion. The same approximation has been made for the reference vectors, whose errors, naturally, are expected to be much smaller. Mathematically, if $\hat{W}_i$ is the true observation vector and $\hat{V}_i$ is the true reference vector then

$$\delta\hat{W}_i = \hat{W}_i - \hat{\bar{W}}_i \qquad \delta\hat{V}_i = \hat{V}_i - \hat{\bar{V}}_i \tag{1}$$

are the errors in $\hat{W}_i$ and $\hat{V}_i$, and the covariance matrices of the sensors may be written in terms of a single parameter for each error vector as

$$\begin{aligned} \langle \delta\hat{W}_i \delta\hat{W}_j^T \rangle &= \sigma_{W_i}^2 \delta_{ij} (I - \hat{\bar{W}}_i \hat{\bar{W}}_i^T) \\ \langle \delta\hat{V}_i \delta\hat{V}_i^T \rangle &= \sigma_{V_i}^2 \delta_{ij} (I - \hat{\bar{V}}_i \hat{\bar{V}}_i^T) \\ \langle \delta\hat{W}_i \delta\hat{V}_i^T \rangle &= 0 \end{aligned} \tag{2}$$

where the angle brackets denote the expectation value and I is the 3×3 identity matrix. The superscript T denotes the matrix transpose. The $\delta\hat{W}_i$ and $\delta\hat{V}_i$ are further assumed to be unbiased to first order in σ_{W_i} and σ_{V_i} §

$$\langle \delta\hat{W}_i \rangle = 0 \qquad \langle \delta\hat{V}_i \rangle = 0 \tag{3}$$

The quantity of interest is the total variance, σ_i^2, given by

$$\sigma_i^2 = \sigma_{W_i}^2 + \sigma_{V_i}^2 \tag{4}$$

In Ref. 1 this was the quantity which proved to be fundamental. This is intuitively satisfying since the reference-vector and observation-vector errors have been assumed to be uncorrelated and an erroneous rotation of the observation vectors will produce the same attitude error as an equal but opposite rotation of the reference vectors. The variance estimator developed in this paper determines σ_i^2.

Section II of this report develops the estimator for the sensor variances. This variance estimator is independent of the deployment of the sensors or an a priori estimate of the sensor accuracies and also independent of any knowledge of the attitude.

Section III presents expressions for the covariances of the estimators under the assumption that the sensor errors have a normal distribution.

In Sec. IV we present specific results for the Magsat mission. In the early part of the Magsat mission there was some uncertainty as to the actual performance of the fine attitude sensors, which was difficult to ascertain because reliable methods were not available for separating attitude errors into their individual sensor components. The analysis of in-flight data presented here removed those ambiguities.

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§In fact, it easy to show that $\langle \delta\hat{W}_i \rangle = -\sigma_{W_i}^2 \hat{\bar{W}}_i$ and similarly for $\langle \delta\hat{V}_i \rangle$.

The sensor-variance estimator assumes that there are no systematic biases in the data; that is, that any effective misalignments of the sensors have been removed. An algorithm for estimating such misalignments, which relies on much the same analysis as the variance estimator, is presented in Sec. V. This method is more efficient and less ambiguous than the methods in current use[2-4] since it does not require an attitude reference and avoids the use of trigonometric functions. The algorithm is applied in Sec. VI to data from the Solar Maximum Mission (SMM) spacecraft.

II. The Variance Estimator

It is assumed that the spacecraft carries at least three vector sensors, the need for which will become apparent in what follows. This situation is not uncommon. Among recent missions both Magsat and the Solar Maximum Mission (SMM) spacecraft carried two fixed-head star trackers and a precision sun sensor. Spacecraft with less demanding attitude accuracy requirements are equipped routinely with a vector magnetometer, sun sensor, and Earth sensor.

The estimator is derived as follows: Select one pair of sensors, say i and j, and examine the quantity

$$z_{ij} = [(\hat{V}_i \cdot \hat{V}_j) - (\hat{W}_i \cdot \hat{W}_j)]^2 + [|\hat{V}_i \times \hat{V}_j| - |\hat{W}_i \times \hat{W}_j|]^2 \tag{5}$$

If there were no errors in the observation or reference vectors, then the reference vectors would be related to the observation vectors by the same orthogonal transformation (the attitude matrix) and z_{ij} would vanish. In fact, substituting Eqs. (1) into the above equation and keeping only lowest order terms leads to

$$z_{ij} = [|\hat{V}_i \times \hat{V}_j|^{-1} \delta(\hat{V}_i \cdot \hat{V}_j) + |\hat{W}_i \times \hat{W}_j|^{-1} \delta(\hat{W}_i \cdot \hat{W}_j)]^2 \tag{6}$$

where δ again denotes the deviation from the true value.

The expectation value of z_{ij} is readily evaluated using Eqs. (1) and (6) and leads to

$$\langle z_{ij} \rangle = \sigma_i^2 + \sigma_j^2 \tag{7}$$

This suggests that we define an estimator

$$\hat{z}_{ij} = \frac{1}{N} \sum_{m=1}^{N} z_{ij}(m) \tag{8}$$

where the sum is over N independent measurements with the sensor pair (i,j). Note that the measurements may be over any set of attitudes and sightings. As N becomes infinitely large, $\hat{z}_{ij}$ assumes the value of the mean as given by Eq. (7). If it is assumed further that the sensor errors are normally distributed (as will be assumed in the next section to compute the covariance of the estimator), then $\hat{z}_{ij}$ is also the maximum likelihood estimator.

If the spacecraft carries three or more vector sensors, then a sufficient number of sensor pairs are available so that the individual sensor variances can be extracted. An example is given in Sec. IV.

The estimator $\hat{z}_{ij}$ has several useful properties. It does not require an a priori estimate of the sensor accuracies and it is also independent of the relative deployment of the sensors. More importantly, it is independent of the spacecraft attitude. Thus there is no source of error from an attitude reference, which must be computed ultimately from the sensor measurements themselves.

As an aside, we may note that the estimator z_{ij} is closely related to the overlap eigenvalue λ_{max} of Ref. 1, which is given by

$$\lambda_{max} = 1 - \min_A \frac{1}{2} \sum_{i=1}^{n} a_i |\hat{W}_i - A\hat{V}_i|^2 \tag{9}$$

where the minimum is taken over all values of the orthogonal matrix A and the non-negative weights have unit sum. If we denote by λ_{ij} the value of λ_{max} for the case when

$$a_i + a_j = 1 \tag{10}$$

and all other weights are chosen to vanish, then it can be shown that to first order in the variances

$$z_{ij} = 2(a_i a_j)^{-1}(1 - \lambda_{ij}) \tag{11}$$

In the Magsat mission software, the quantity $(1-\lambda_{ij})$ is obtained as intermediate output and it is this quantity which has been used in the analysis presented in Sec. IV. Note that z_{ij} should be independent of the specific values of a_i and a_j so long as neither vanishes and the two sum to unity. This was, in fact, observed to be the case within known systematic errors.

Equations (11) and (7) provide a means for estimating the expectation value of the overlap eigenvalue, namely,

$$\langle \lambda_{ij} \rangle = 1 - \tfrac{1}{2} a_i a_j (\sigma_i^2 + \sigma_j^2) \tag{12}$$

This result can be generalized for an arbitrary number of sensors, although it is not as simple in form. The general result is

$$\langle \lambda \rangle = 1 - \sum_{i=1}^{n} a_i \sigma_i^2 + \frac{1}{2} \sum_{i=1}^{n} a_i^2 \sigma_i^2 \mathrm{Tr}[(1 - \hat{W}_i \hat{W}_i^T) M^{-1}] \tag{13}$$

where

$$M = \sum_{i=1}^{n} a_i (1 - \hat{W}_i \hat{W}_i^T) \tag{14}$$

This reduces to Eq. (12) for the special case $n = 2$.

III. Covariance Analysis

If, in addition to Eqs. (2) and (3), it is assumed that the $\delta\hat{W}_i$ and the $\delta\hat{V}_i$ have a normal distribution, it is possible to compute higher moments of the random variable z_{ij}. The computation is straightforward and yields

$$\mathrm{Var}(z_{ij}) = 2(\sigma_i^2 + \sigma_j^2)^2 \tag{15}$$

$$\mathrm{Cov}(z_{ij}, z_{ik}) = 2\sigma_i^4 \cos^2\theta_{ijk} \tag{16}$$

$$\mathrm{Cov}(z_{ij}, z_{km}) = 0 \tag{17}$$

where

$$\cos\theta_{ijk} = \frac{(\hat{W}_i \times \hat{W}_j)}{|\hat{W}_i \times \hat{W}_j|} \cdot \frac{(\hat{W}_i \times \hat{W}_k)}{|\hat{W}_i \times \hat{W}_k|} \tag{18}$$

The variances and covariances of the estimator are identical to the above expressions except for an additional factor $1/N$. For simplicity $\hat{W}_i$ has been written in place of $\bar{W}_i$.

IV. Application to Magsat

The Magsat spacecraft was equipped with two fixed-head star trackers (manufactured by Ball Brothers Research Corporation) denoted here by FHST1 and FHST2 and a precision sun sensor (manufactured by the Adcole Corporation) denoted by FSS. The accuracies specified before launch by the manufacturers were

$$\sigma_{FHST1} = 8 \text{ arc-sec}$$
$$\sigma_{FHST2} = 7 \text{ arc-sec} \tag{19}$$
$$\sigma_{FSS} = 12 \text{ arc-sec}$$

These sensors were mounted in the spacecraft with boresights directed approximately along the vectors

$$\hat{W}_1 = \left(\sqrt{3/8}, \sqrt{3/8}, ½\right)^T$$
$$\hat{W}_2 = \left(-\sqrt{3/8}, \sqrt{3/8}, ½\right)^T \quad (20)$$
$$\hat{W}_3 = (0,0,1)^T$$

where the subscripts *1, 2, 3* follow the same order as the literal subscripts of Eqs. (19). The definition of the spacecraft-fixed coordinate system is unimportant to the present discussion.

Since the observations were always close to the boresight, the boresight vectors may be used to compute the correlation angles of Eq. (18) with the results

$$\cos^2\theta_{312} = 0 \qquad \cos^2\theta_{123} = \cos^2\theta_{213} = 1/5 \quad (21)$$

The estimators for the variances of the individual sensor errors are given in terms of the estimators $\hat{Z}_{ij}$ by

$$\hat{S}_1^2 = (\hat{Z}_{12} + \hat{Z}_{13} - \hat{Z}_{23})/2$$
$$\hat{S}_2^2 = (\hat{Z}_{12} + \hat{Z}_{23} - \hat{Z}_{13})/2 \quad (22)$$
$$\hat{S}_3^2 = (\hat{Z}_{13} + \hat{Z}_{23} - \hat{Z}_{12})/2$$

and the associated variances and covariances are given by

$$\mathrm{Var}(\hat{S}_1^2) = (1/N)\{\sigma_1^4(1+\cos^2\theta_1) + \sigma_2^4\sin^2\theta_2$$
$$+\sigma_3^4\sin^2\theta_3 + \sigma_1^2\sigma^2_2 + \sigma_1^2\sigma_3^2 + \sigma_2^2\sigma_3^2\} \quad (23a)$$

$$\mathrm{Cov}(\hat{S}_1^2,\hat{S}_2^2) = (1/N)\{\sigma_1^2\sigma_2^2 - \sigma_1^2\sigma_3^2 - \sigma_2^2\sigma_3^2 - \sigma_3^4\sin^2\theta_3\} \quad (23b)$$

For simplicity θ_i has been written in place of θ_{ijk}. For a system with only three sensors, no ambiguity can result. The variances and covariances of the remaining variables can be obtained by cyclic permutation of the indices. Note that although the quantities σ_i^2 and σ_j^2 are surely independent, the respective estimators $\hat{S}_i^2$ and $\hat{S}_j^2$ are not.

Approximately 100 samples of Magsat data were analyzed. The post-launch results for the standard deviations using Eqs. (22) and (23a) are

$$\sigma_{\mathrm{FHST1}} = 9.2 \pm 1.2 \text{ arc-sec}(1\sigma)$$
$$\sigma_{\mathrm{FHST2}} = 8.0 \pm 1.4 \text{ arc-sec}(1\sigma) \quad (24)$$
$$\sigma_{\mathrm{FSS}} = 11.2 \pm 1.0 \text{ arc-sec}(1\sigma)$$

in close agreement with the manufacturers' specifications. Note that the most accurate sensor need not have the most accurately determined variances by this method.

V. Estimation of Attitude Sensor Misalignments

The preceding analysis for estimating the variance of the attitude sensor errors assumed that a consistent set of sensor alignments was available. A necessary and sufficient condition for this to be true is that

$$\langle \hat{W}_i \cdot \hat{W}_j \rangle = \langle \hat{V}_i \cdot \hat{V}_j \rangle \quad (25)$$

for every sensor pair. A consistent set of alignments may be very far from the truth, however, since the substitution

$$\hat{W}_i \to \hat{W}_i' = T\hat{W}_i \qquad (i = 1,\ldots,n) \quad (26)$$

where T is an arbitrary orthogonal matrix (independent of i), does not affect Eq. (25).

For in-flight estimation the matrix T is indeterminate. This indeterminancy can be removed by choosing a subset of the sensor measurements as primary, using these to establish an attitude reference, and then using regression techniques to determine the misalignments associated with the remaining measurements. This is the procedure which was followed on Magsat.[3] We develop in this section a superior technique which treats all the sensors equally and effectively chooses T to minimize the overall deviation of the sensor alignments from their a priori values determined before launch.

Let $\hat{U}_i$ be the observation unit vector measured by the ith sensor and resolved along sensor axes. This is related to the observation unit vector in the spacecraft-body coordinate system by the alignment matrix according to

$$\hat{W}_i = S_i\hat{U}_i \quad (27)$$

The unit vector $\hat{W}_i$ is corrupted by misalignment errors which arise from imperfect knowledge of S_i. The true value of the body-referenced observation unit vector following the alignment calibration is

$$\hat{W}_i^{\mathrm{cal}} = M_i\hat{W}_i \quad (28)$$

which satisfies Eq. (25). This suggests that the misalignment matrices be chosen to minimize the loss function

$$L(\mathrm{post}) = \frac{1}{2}\sum_{i,j}^{n}{}' \, b_{ij}\,|\hat{W}_i \cdot M_i^T M_j \hat{W}_j - \hat{V}_i \cdot \hat{V}_j|^2 \quad (29)$$

where the designation "post" denotes that only post-launch data are used. The prime on the summation denotes that the terms with $i=j$ are excluded.

Within the spirit of maximum likelihood estimation the weight b_{ij} is chosen to be

$$b_{ij} = \langle |\hat{W}_i^{\mathrm{cal}} \cdot \hat{W}_j^{\mathrm{cal}} - \hat{V}_i \cdot \hat{V}_j|^2 \rangle^{-1} \quad (30)$$

which is readily evaluated using Eqs. (2) to give

$$b_{ij} = \{ |\hat{W}_i \times \hat{W}_j|^2 (\sigma_i^2 + \sigma_j^2) \}^{-1} \quad (31)$$

(In general, for ease of notation, we will ignore the distinction between $\hat{W}_i^{\mathrm{cal}}$, $\hat{W}_i$, $\langle \hat{W}_i \rangle$, etc., in final expressions when the difference is not of numerical importance.)

Equation (29) is the loss for a single set (frame) of measurements. For N frames of measurements this becomes¶.

$$L(\mathrm{post}) = \frac{1}{2}\sum_{m=1}^{N}\sum_{i,j}^{n}{}' \, b_{ij}(m)\,|\hat{W}_i(m)$$
$$\cdot M_i^T M_j \hat{W}_j(m) - \hat{V}_i(m) \cdot \hat{V}_j(m)|^2 \quad (32)$$

The sought-for M_i minimize the loss function of Eq. (32). These M_i are not absolutely determinable since only quotients of the M_i appear. For the moment, we note that the a priori values of the M_i are simply the identity matrix, and the a posteriori values will correspond to small rotations and can be approximated by the expression

$$M_i = I + [\underline{\theta}_i] \quad (33)$$

¶Note: When data from either sensor i or sensor j is not available in frame m, then we understand $b_{ij}(m) = 0$. Otherwise, it is given by Eq. (43).

where

$$[\underline{n}] = \begin{bmatrix} 0 & n_3 & -n_2 \\ -n_3 & 0 & n_1 \\ n_2 & -n_1 & 0 \end{bmatrix} \tag{34}$$

Substituting this expression into Eq. (32) leads to

$$L(\text{post}) = \frac{1}{2}\sum_{m=1}^{N}\sum_{i,j}^{n}{}' b_{ij}(m)\,|\hat{W}_i(m)\cdot\hat{W}_j(m) - \hat{V}_i(m)\cdot\hat{V}_j(m) - [\hat{W}_i(m)\times\hat{W}_j(m)]\cdot(\theta_i-\theta_j)|^2 \tag{35a}$$

The a priori values of the θ_i are simply 0.

The indeterminancy in the optimal set of θ_i is removed by adding to the a posteriori loss function of Eq. (35a) an a priori loss function

$$L(\text{prior}) = \frac{1}{2}\sum_{i,j}^{n}\theta_i^T[P(-)^{-1}]_{ij}\theta_j \tag{35b}$$

where $P(-)$ is the $3n\times 3n$ covariance matrix of the initial alignment calibration. As a rule, it will be assumed for the ground calibration that the cross-covariance submatrices vanish so that

$$[P(-)^{-1}]_{ii} = [P(-)_{ii}]^{-1} \tag{36a}$$

$$[P(-)^{-1}]_{ij} = 0 \qquad (i\neq j) \tag{36b}$$

[The need for an a priori estimate of the misalignments when the attitude is unknown has not always been recognized. For example, des Jardins[6] argues incorrectly that for a sufficiently large number of measurements the $3n$ sensor misalignment angles are overdetermined and hence can be solved for. He neglects, however, the indeterminancy introduced by Eq. (26).]

The total loss function

$$L = L(\text{prior}) + L(\text{post}) \tag{37}$$

is now minimized with respect to the θ_i. This leads straightforwardly to the equations

$$\{[P(-)^{-1}]_{ii}+F_i\}\theta_i + \sum_{j}^{n}{}'\{[P(-)^{-1}]_{ij}-G_{ij}\}\theta_j = H_i \tag{38}$$

with

$$G_{ij} = \sum_{m=1}^{N} b_{ij}(m)\{(\hat{W}_i\times\hat{W}_j)(\hat{W}_i\times\hat{W}_j)^T\}_m \tag{39a}$$

$$F_i = \sum_{j}^{n}{}' G_{ij} \tag{39b}$$

$$H_i = \sum_{m=1}^{N}\sum_{j}^{n}{}' b_{ij}(m)\{(\hat{W}_i\cdot\hat{W}_j - \hat{V}_i\cdot\hat{V}_j)(\hat{W}_i\times\hat{W}_j)\}_m \tag{39c}$$

which may be solved directly for the θ_i.

The covariance and cross-covariance submatrices of the final $3n\times 3n$ covariance matrix may be read directly from Eq. (38) as

$$[P(+)^{-1}]_{ii} = [P(-)^{-1}]_{ii} + F_i \tag{40a}$$

$$[P(+)^{-1}]_{ij} = [P(-)^{-1}]_{ij} - G_{ij} \qquad (i\neq j) \tag{40b}$$

Note that G_{ij}, F_i, and H_i are roughly proportional to N.

Alternatively, the M_i could have been determined using an a posteriori loss function of the form

$$L(\text{post}) = \frac{1}{2}\sum_{m=1}^{N}\sum_{i,j}^{n}{}' b'_{ij}(m)\{\hat{W}_i\cdot M_i^T M_j\hat{W}_j - \hat{V}_i\cdot\hat{V}_j]^2 + [\,|\hat{W}_i\times M_i^T M_j\hat{W}_j| - |\hat{V}_i\times\hat{V}_j|\,]^2\}_m \tag{41}$$

which is more similar to Eq. (5) than is the expression in Eq. (32). This is also equivalent to choosing the misalignments to maximize the overlap eigenvalue of Eq. (9). As it turns out, use of this a posteriori loss function in a maximum likelihood estimate of the misalignment angles leads to a result which is exactly identical to Eqs. (38-40) (with $b_{ij}(m)$ unchanged) with the sole exception that H_i given by Eq. (39c) is replaced by H'_i given by

$$H'_i = \sum_{m=1}^{N}\sum_{j}^{n}{}' b_{ij}(m)\{[(\hat{W}_i\cdot\hat{W}_j)\,|\hat{V}_i\times\hat{V}_j| - |\hat{W}_i\times\hat{W}_j|(\hat{V}_i\cdot\hat{V}_j)]\,|\hat{W}_i\times\hat{W}_j|(\hat{W}_i\times\hat{W}_j)\}_m \tag{39c'}$$

The two vectors H_i and H'_i are equal to first order in the misalignment angles and are, therefore, interchangeable. Equation (39c) imposes a slightly reduced computational burden.

Given the θ_i, the corrected alignment matrix is given by

$$S_i^{\text{corrected}} = M_i S_i = M(\theta_i) S_i \tag{42}$$

where $M(\theta_i)$ is exactly orthogonal. A convenient expression for $M(\theta)$ is obtained by interpreting θ as twice the Gibbs vector of the misalignment rotation. Then

$$M(\theta) = (I+|g|^2)^{-1}\{(I-|g|^2)I + 2gg^T + 2[\underline{g}]\} \tag{43}$$

and

$$g = \theta/2 \tag{44}$$

The expression given by Eq. (43) is exactly orthogonal and consistent with Eq. (33).

Once the new alignment matrix has been computed, these procedures may be iterated as often as necessary to achieve the desired accuracy.

VI. Application to SMM

The algorithm developed in the previous section has been applied to computing misalignments of the fine attitude sensors of the Solar Maximum Mission (SMM) spacecraft. The choice of spacecraft was dictated by the availability of an active attitude ground support system in the summer of 1981. The demise of Magsat occurred during the previous summer and attitude ground support had already been terminated.

The sensor accuracies for the SMM spacecraft for the two fixed-head star trackers (FHST1 and FHST2) and the fine-pointing sun sensor (FPSS) were given as

$$\sigma_{\text{FHST1}} = 15 \text{ arc-sec} \tag{45a}$$

$$\sigma_{\text{FHST2}} = 10 \text{ arc-sec} \tag{45b}$$

$$\sigma_{\text{FPSS}} = 10 \text{ arc-sec} \tag{45c}$$

The a priori covariance matrix of the misalignments is composed of two components: the accuracy of the ground calibration, which can be expected to lead to small errors, and the errors introduced by launch shock, thermal stresses, zero gravity, etc., which can be expected to be large. For the three sensors the a priori alignment covariances were taken to be

$$P^{\S}_{\text{FHST1}}(-) = \text{Diag}(60^2, 10^2, 10^2)\ (\text{arc-sec})^2 \tag{46a}$$

$$P^S_{FHST2}(-)=\text{Diag}(60^2, 10^2, 10^2)\ (\text{arc-sec})^2 \tag{46b}$$

$$P^S_{FPSS}(-)=\text{Diag}(5^2, 30^2, 30^2)\ (\text{arc-sec})^2 \tag{46c}$$

where Diag denotes a diagonal matrix and the superscript 2 in the argument denotes the square. The order of the diagonal elements is the boresight followed by the two sensor axes. The origin of these values is not important to this paper. Note that these a priori values are given in the individual sensor frames and must be transformed to the body frame before implementing the results of the previous section.

The results for the misalignments (in the *body* frame) were found to be

$$\theta_{FHST1}=(14.8, 20.1, -4.5)^T \text{ arc-sec} \tag{47a}$$

$$\theta_{FHST2}=(4.4, -5.6, 1.8)^T \text{ arc-sec} \tag{47b}$$

$$\theta_{FPSS}=(0.0, -5.1, -3.8)^T \text{ arc-sec} \tag{47c}$$

with predicted standard deviations given by

$$\sigma_{FHST1}(+)=(35.0, 44.0, 16.0)^T \text{ arc-sec} \tag{48a}$$

$$\sigma_{FHST2}(+)=(35.0, 44.0, 16.0)^T \text{ arc-sec} \tag{48b}$$

$$\sigma_{FPSS}(+)=(5.0, 27.0, 11.0)^T \text{ arc-sec} \tag{48c}$$

A statistical analysis of these results is not possible since there is only a single sample of the launch-induced errors. However, it should be noted that every one of the misalignments calculated lies well within the a priori estimate of the misalignment standard deviations, while naively we should expect only two-thirds of the misalignments to lie within that limit. This indicates that our estimates of the errors induced by launch shock are probably too pessimistic. Crudely, this could be remedied by reducing all the a priori standard deviations by a common factor so that one-third fall outside the a priori limit. Such a remedy is hard to justify formally but the estimated alignment accuracies which follow from it may be more realistic. The actual values of the sensor misalignments, however, would not be expected to be much affected by such a post hoc artifice. This is because, for a large enough sample, the sensor misalignments will be determined up to a common orthogonal transformation by the observations alone. The common orthogonal transformation is determined from the a priori loss function and its value should be unaffected as long as the relative values of the a priori covariances are not changed.

The a posteriori values for the misalignment standard deviations given in Eqs. (48) are dominated by the a priori values. This can be seen from an examination of the random errors in the misalignment calibration, for which the standard deviations were found to be smaller than the a pirori values by a factor of at least 2 in every case. Hence it is probably a good rule of thumb that estimates of the misalignments are more meaningful than estimates of the alignment accuracies, which are not truly measurable.

VII. Discussion and Conclusions

Simple estimators have been developed for determining inflight the accuracies and alignments of vector attitude sensors. The estimators are unbiased and independent of the configuration of the attitude sensors onboard the spacecraft. Their implementation does not require an attitude reference, which ultimately must be computed from the sensor measurements themselves. These estimators are robust and computationally efficient and have been exercised using inflight data from Magsat and SMM.

The particular advantage which these two algorithms offer over other batch least-squares estimators or Kalman filters is that they are relatively insensitive to many of the system variables, use the least amount of data necessary, and are mechanized to minimize the computations. Misalignments and variances can, of course, always be computed in a Kalman filter[2,4,7] or in a more elaborate batch least-squares estimator[2,3,5] along with the attitude and other spacecraft variables. The disadvantages of this latter approach are threefold: First, treating such a large augmented state vector in the Kalman filter, rather than partitioning the problem into mutually independent estimates of attitude, misalignments, and variances, entails a much greater computational burden, which would be appropriate to analytical studies but very uneconomical in production software. Second, although theoretically the accuracy of the estimate will increase as more variables are estimated, an increase in the number of variables estimated also increases the number of unmodeled processes to which each variable is connected. Hence the increased accuracy may prove to be elusive (see Ref. 7, for example). Last, very simple estimators, such as the ones presented here, have the advantage of great portability. By contrast, very elaborate estimation software is difficult to transfer from one system to another.

As an example of the computational saving, the batch least-squares software used by Magsat to estimate sensor misalignments,[3] which used whole angles and required the computation of the attitude, required several hours of execution time. The present algorithm, using similar amounts of data, required much less than a minute.

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