PHYSICAL REVIEW C VOLUME 13, NUMBER 6 JUNE 1976

Nonrelativistic hard-pion production and current-field algebra. II. Reactions with composite targets*

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(Received 11 August 1975)

The theory of hard-pion production in nucleon-nucleon collision of the authors has been extended to nucleon-nucleus collision. The theory is based on making the leading corrections to the soft-pion limit. Thus it is an approximate theory, but is probably superior to any perturbation theory.

[NUCLEAR REACTIONS Theory, pion production, *N*-*N* collisions, soft-pion theory, current-field algebra.]

I. INTRODUCTION

In recent years an enormous amount of work has been done on pion production in nucleon-nucleon and nucleon-nucleus collisions. In most of these works the pion-production operator, to be used with nonrelativistic nuclear wave functions, have been written down in an *ad hoc* manner without sufficient justification. In fact, there is some debate whether the operator should involve the pion momentum or the pion-nucleon relative momentum. A few years back we studied[1] the problem of pion production in nucleon-nucleon collision and developed the pion-production operator to be used with a nonrelativistic nucleon-nucleon wave function. The theory is approximate but not perturbative. The final-state interaction of the pion is ignored for simplicity. Thus the theory is valid only near the threshold. The dynamics is introduced through current-field algebra which enables us to evaluate an unambiguous soft-pion limit. The hard-pion result is obtained by using the mass dispersion relation and retaining the leading cuts. Although there have been other similar works[2,3] on pion production in *N*-*N* collision, the problem of pion production in nucleon-nucleus collision has not received any attention along these lines. In this paper we extend our previous method to this problem and derive the pion-production operator.

The transition amplitude for pion production in the reaction $i \to \pi + f$ is given by

$$\mathfrak{M} = \langle f | j_\pi^a(0) | i \rangle = [m_\pi{}^2 - (p_f - p_i)^2] \langle f | \phi_\pi^a(0) | i \rangle , \qquad (1.1)$$

where ϕ_π^a and j_π^a are the pion field and current, respectively, and a is the component of the isospin. The equality is understood in the limit that $(p_f - p_i)^2$ tends toward $m_\pi{}^2$. Naively, using the strong version of partially conserved axial-vector current (PCAC), which relates the pion field to the divergence of the axial-vector current according to

$$\partial_\mu A_\mu^a = \frac{f_\pi}{\sqrt{2}} \phi_\pi^a ,$$

the amplitude becomes

$$\frac{\sqrt{2}}{f_\pi} [m_\pi{}^2 - (p_f - p_i)^2](p_f - p_i)_\mu \langle f | A_\mu^a(0) | i \rangle . \qquad (1.2)$$

An on-shell extrapolation (i.e., all particles other than the pion are on the mass shell) amounts to calculating Eq. (1.2) in the limit that $p_f - p_i \to 0$. This is the extrapolation method of Adler[4] and Adler and Dothan.[5] That the corrections to this limit can be very large is made evident from the fact that the result of the extrapolation differs greatly depending on whether p_i or p_f is fixed, since the projectile-target interactions are certainly very different at 0 and 140 MeV.

To avoid this ambiguity we have reduced an additional particle from the amplitude in Eq. (1.1) and written a mass dispersion relation in the style of Fubini and Furlan.[6] As in the previous work[1] we show that by keeping only the leading cuts this mass dispersion relation can be related to the Lippmann-Schwinger equation of nonrelativistic potential theory.

This program allows us to identify unambiguously the nonrelativistic potential for pion production which must be used in a more complete calculation including the final-state interaction of the pion with the nuclear system. This last interaction is not examined in the present work.

At present the production of pions from nuclear targets has received renewed attention due largely to the existence for several years of accurate experimental information.[7] Several attempts[8] have been made to explain the detailed spectroscopic results without sufficient regard for the nature of the production operator. The present work attempts to remove this lacuna.

A program not unrelated to the one presented here has done much to clarify the elastic scattering and photoproduction of pions on nuclei. These studies are described fully in the review article of M. Ericson and M. Rho,[9] which gives a capsule account of the work of the present authors as well.

In Sec. II we derive the soft-pion limit. The next section contains the result for the hard pion which is the objective of this paper. In the last section we discuss the difference between the soft-pion and the hard-pion results.

II. SOFT-PION LIMIT

In this and in succeeding sections we follow the method of the previous work.[1] The S matrix for the process

$$N(p_1, s) + Z^A \to \pi^a(k) + f, \tag{2.1}$$

in which a nucleon of four-momentum p_1 and spin projection s impinges on a target nucleus Z^A (with momentum p_2) to produce a pion of isospin a ($a = 1, 2, 3$) and four-momentum k and some unspecified configuration of particles f, is given by[10]

$$S = -i(2\pi)^4\delta^4(p_f + k - p_1 - p_2)\langle f\pi^a(k)|\bar{j}(0)|Z^A\rangle \times u(p_1, s), \tag{2.2}$$

where $\bar{j}(0)$ is the adjoint nucleon current operator

$$\bar{j}(x) = \bar{\psi}(x)(\overleftarrow{-i\not\nabla - M}), \tag{2.3}$$

with $\bar{\psi}(x)$ the adjoint canonical nucleon field. We suppress generally all isospin indices for the nucleon as well as the "in" and "out" specification of the Heisenberg states. As in the previous work, we work in the rest frame of the pion ($k_0 = m_\pi$, $\vec{k} = \vec{0}$).

Within the framework of the Lehmann-Symanzik-Zimmerman (LSZ) formalism, the matrix element appearing in Eq. (2.2) may be rewritten as

$$\begin{aligned}\langle f\pi^a(k)|\bar{j}(0)|Z^A\rangle u(p_1, s) &= i\int d^4x\, e^{ik\cdot x}(\Box + m_\pi^2)\langle f|\,T[\phi^a(x)\bar{j}(0)]\,|Z^A\rangle u(p_1, s)\\ &= \frac{i\sqrt{2}}{f_\pi}\int d^4x\, e^{ik\cdot x}(\Box + m_\pi^2)\langle f|\,T[D^a(x)\bar{j}(0)]\,|Z^A\rangle \times u(p_1, s) \qquad (2.4)\\ &= F(k), \qquad (2.5)\end{aligned}$$

where $\phi^a(x)$ is the canonical pion field $D^a(x)$ is the divergence of the axial-vector current

$$D^a(x) = \partial_\mu A_\mu^a(x), \tag{2.6}$$

and f_π is the pion weak-decay constant whose numerical value is $0.93m_\pi^3$. In passing from Eq. (2.4) to Eq. (2.5) we have used the divergence of the axial-vector current as an interpolating field for the pion. On the pion mass shell, $k^2 = m_\pi^2$, this substitution is rigorous; otherwise, it may be thought of as defining the analytic continuation of the reaction amplitude for unphysical pion momenta. The constant f_π is determined by

$$\langle\pi^+|D^+(0)|0\rangle = f_\pi .$$

(In the previous work[1] the amplitude corresponding to Eq. (2.4) was written with a retarded commutator rather than with a chronological product. However, in making the connection with the nonrelativistic theory in the succeeding sections—in particular, in treating the left-hand cut—it will be more convenient to retain the chronological product.)

In this way we generalize $F(k)$ to a function of an arbitrary four-momentum q, which we chose to have always vanishing spatial components (pion rest frame), and we define with Fubini and Furlan[6] the quantities

$$M_0(q_0) = \frac{\sqrt{2}}{f_\pi}\int d^4x\, e^{iq_0x}(\Box + m_\pi^2)\langle f|\,T[A_0^a(x)\bar{j}(0)]\,|Z^A\rangle u(p_1, s), \tag{2.7a}$$

$$R(q_0) = \frac{-i\sqrt{2}}{f_\pi}\int d^4x\, e^{iq_0x}(\Box + m_\pi^2)\langle f|[A_0^a(x), \bar{j}(0)]\,\delta(x_0)|Z^A\rangle u(p_1, s), \tag{2.7b}$$

which satisfy for all q_0 the identity

$$F(q_0) = R(q_0) + q_0M_0(q_0). \tag{2.8}$$

The equal-time commutator

$$[A_0^a(x), \bar{j}(0)]\delta(x_0)$$

has been discussed in detail in the previous work[1] and in references cited therein. Apart from possible Schwinger terms, which do not contribute to Eq. (2.7b), it was shown that

$$[A_0^a(x), \bar{j}(0)]\,\delta(x_0) = [2M\bar{\psi}(0) + j(0)]\gamma_5\tfrac{1}{2}\tau^a\delta^{(4)}(x). \tag{2.9}$$

Equation (2.7b) thus becomes

$$R(q_0) = -i\frac{\sqrt{2}}{f_\pi}(m_\pi^2 - q_0^2)\langle f|\bar{j}(0)|Z^A\rangle \frac{\not{p}_f - \not{p}_2 + M}{\not{p}_f - \not{p}_2 - M}\gamma_5 \tfrac{1}{2}\tau^a u(p_1, s)\,.$$

Recalling momentum conservation we rewrite $R(q_0)$ as

$$R(q_0) = -i\frac{\sqrt{2}}{f_\pi}(m^2 - q^2)\langle f|\bar{j}(0)|Z^A\rangle \frac{\not{p}_1 - \not{k} + M}{\not{p}_1 - \not{k} - M}\gamma_5 \tfrac{1}{2}\tau^a u(p_1, s)\,. \tag{2.10}$$

The structure of $M_0(q_0)$ is more complicated. Inserting a complete set of states in the commutator of Eq. (2.7a), we obtain

$$M_0(q_0) = M_0^{\mathrm{I}}(q_0) + M_0^{\mathrm{II}}(q_0)\,,$$

with

$$M_0^{\mathrm{I}}(q_0) = \frac{-i(2\pi)^3\sqrt{2}(m_\pi^2 - q_0^2)}{f_\pi}\sum_{|n\rangle}\langle f|A_0^a(0)|n\rangle\langle n|\bar{j}(0)|Z^A\rangle u(p_1, s)\frac{\delta^{(3)}(\vec{p}_f - \vec{p}_n)}{E_f + q_0 - E_n + i\epsilon}\,, \tag{2.11a}$$

$$M_0^{\mathrm{II}}(q_0) = \frac{-i(2\pi)^3\sqrt{2}(m_\pi^2 - q_0^2)}{f_\pi}\sum_{|n\rangle}\langle f|\bar{j}(0)|n\rangle\langle n|A_0^a(0)|Z^A\rangle u(p_1, s)\frac{\delta^{(3)}(\vec{p}_2 - \vec{p}_n)}{E_2 - q_0 - E_n + i\epsilon}\,. \tag{2.11b}$$

It should be noted that for physical pion momentum ($q_0 = m_\pi$) $R(q_0)$ vanishes and the physical amplitude $F(m_\pi)$ is given by $M_0(m_\pi)$ (understood as the limiting value as $q_0 \to m_\pi$). In the limit that the pion four-momentum vanishes ($q_0 = 0$), however, the structure of the amplitude is much different. In this case, $R(0)$ is nonvanishing with

$$R(0) = \frac{-i\sqrt{2}m_\pi^2}{f_\pi}\langle f|\bar{j}(0)|Z^A\rangle \frac{\not{p}_1 - \not{k} + M}{\not{p}_1 - \not{k} - M}\gamma_5 \tfrac{1}{2}\tau^a u(p_1, s)\,, \tag{2.12}$$

and it is the limit of $q_0 M_0(q_0)$ as $q_0 \to 0$ which contributes, that is, the residue of $M_0(q_0)$ at $q_0 = 0$. This may be calculated very simply. The result is

$$\lim_{q_0\to 0} q_0 M_0^{\mathrm{I}}(q_0) = -i\frac{\sqrt{2}m_\pi^2}{f_\pi}\sum_{|n\rangle}\langle f_n|A_0^a(0)|f_n'\rangle\langle f'|\bar{j}(0)|Z^A\rangle u(p_1, s)\,, \tag{2.13a}$$

$$\lim_{q_0\to 0} q_0 M_0^{\mathrm{II}}(q_0) = +i\frac{\sqrt{2}m_\pi^2}{f_\pi}\langle f|\bar{j}(0)|Z^A\rangle u(p_1, s)\langle Z^A|A_0^a(0)|Z^A\rangle\,, \tag{2.13b}$$

where we have written $|f\rangle = |f_1 f_2 \cdots f_n \cdots\rangle$ in terms of the stable particles composing f and have summed implicitly over any spin or isospin degrees of freedom in the intermediate states.

For spinless targets the expectation value of the axial-vector current vanishes due to the requirements of parity and, therefore,

$$\lim_{q_0\to 0} q_0 M_0^{\mathrm{II}}(q_0) = 0\,.$$

When the expectation value of $A_0^a(0)$ does not vanish, it is proportional to the velocity of the particle. Hence, at threshold (or very near to threshold) $\langle f_n|A_0^a(0)|f_n'\rangle$ vanishes (or is very small). (Recall that f_n and f_n' are degenerate in energy.) This implies

$$\lim_{q_0\to 0} q_0 M_0^{\mathrm{I}}(q_0) = 0$$

at threshold. Thus, at threshold the soft-pion limit for the reaction amplitude is given by $R(0)$ alone and

$$F(0) = -i\frac{\sqrt{2}m_\pi^2}{f_\pi}\langle f|\bar{j}(0)|Z^A\rangle \times \frac{\not{p}_1 - \not{k} + M}{\not{p}_1 - \not{k} - M}\gamma_5 \tfrac{1}{2}\tau^a u(p_1, s)\,. \tag{2.14}$$

Noting that in Eq. (2.14)

$$\frac{\not{p} - \not{k} + M}{\not{p} - \not{k} - M} = \frac{\not{p} + M}{2p_0 k}\not{k} + O(m_\pi^2/M^2)\,,$$

we write Eq. (2.14) near threshold as

$$F(0) \approx \frac{-i\sqrt{2}m_\pi^2}{f_\pi}\langle f|\bar{j}(0)|Z^A\rangle \frac{\not{p}_1 + M}{2p\cdot k}\not{k}\gamma_5 \tfrac{1}{2}\tau^a u(p_1, s)\,. \tag{2.15}$$

Equation (2.15) is our soft-pion result for pion production near threshold by a spin-zero target. It may be noted that this is a well-defined result and does not suffer from the ambiguities of the Adler-Dothan[4,5] type soft-pion limit. We hasten to add that this remark is not intended to imply that (2.15) is an acceptable result. In the next section we derive the hard-pion result $F(m)$, and discuss the difference between the two results.

III. HARD-PION AMPLITUDE

We now turn to the problem of deriving an approximate expression for the amplitude for production of a physical pion near threshold. The derivation follows very closely the method used in Ref. 1. Apart from using the Low expansion of the amplitude $F(q_0)$, introduced in Eq. (2.5) and its soft-pion limit, $F(0)$, we will assume the existence of a nonrelativistic theory of pion production where the pion-production operator is treated in Born approximation.

Specifically we assume that if $|f\rangle$ and $|n\rangle$ are two *nuclear* states (i.e., they contain no pion), we may write

$$(2\pi)^3\delta^{(3)}(\vec{p}_f+\vec{k}-\vec{p}_n)\langle f|j_\pi^a(0)|n\rangle=(\Psi_f^{(-)}|V_{\vec{k}}^a|\Psi_n^{(+)}), \tag{3.1}$$

where $\Psi_f^{(-)}$ is the exact nonrelativistic Schrödinger wave function for $|f\rangle$, which is the product of two wave functions, the wave function in the c.m. frame and a plane wave describing the motion of the center of mass of the entire system with momentum $\vec{p}_f$ and similarly for $\Psi_n^{(+)}$. The superscript $(+)$ or $(-)$ designates outgoing- or incoming-wave boundary conditions for the relative motion in the appropriate manner. We further assume that the operator $V_{\vec{k}}^a$ is a single-nucleon operator. Since in the present paper we restrict our study to pion production at threshold we disregard the final-state interaction of the pion. Hence we imply that the operator $V_{\vec{k}}^a$ is of the form

$$V_{\vec{k}}^a=\frac{1}{(2\pi)^3}\int d^3\vec{x}\,e^{-i\vec{k}\cdot\vec{x}}\psi_{NR}^\dagger(\vec{x})\Omega^a\psi_{NR}(\vec{x}), \tag{3.2}$$

where $\psi_{NR}(x)$ is the nonrelativistic fermion field and Ω^a is some operator. In the Schrödinger representation

$$V_{\vec{k}}^a=\sum_i e^{-i\vec{k}\cdot\vec{r}_i}V_0^{a,i}, \tag{3.3}$$

where the sum extends over all the nucleons present and $V_0^{a,i}$ is a single-nucleon operator. Our object here is to obtain an expression for $V_{\vec{k}}^a$ valid at least for those states $|n\rangle$ which asymptotically consist of a nucleon and the target Z^A in its ground or some excited state moving towards each other. Since we need to exploit the soft-pion result we work in the pion rest frame and obtain an expression for V_0^a. From this the expression for $V_{\vec{k}}^a$ may be written down using Eq. (3.3).

In the pion rest frame Eq. (3.1) reads

$$(2\pi)^3\delta^{(3)}(\vec{p}_f-\vec{p}_n)\langle f|j_\pi^a(0)|n\rangle=(\Psi_f^{(-)}|V_0^a|\Psi_n^{(+)}). \tag{3.4}$$

The states $|n\rangle$ of interest are composed asymptotically of a nucleon with momentum $\vec{p}_1$ and a system of A nucleons of total momentum $\vec{p}_2$. This subsystem may be in the ground state of the target nucleus but in the derivation which follows we must extend our consideration to the complete set of states of these A nucleons. In general, only the momentum $\vec{p}_2$ of this subsystem will be written explicitly. The equation reads

$$(2\pi)^3\delta^{(3)}(\vec{p}_f-\vec{p}_1-\vec{p}_2)\langle f|j_\pi^a(0)|p_1p_2(in)\rangle=(\Psi_f^{(-)}|V_0^a|\Psi^{(+)}(\vec{p}_1,\vec{p}_2)). \tag{3.5}$$

The states $|\Psi_f^{(+)})$, $|\Psi^{(+)}(\vec{p}_1,\vec{p}_2))$, etc., are eigenstates of a nuclear Hamiltonian

$$H=T+U, \tag{3.6}$$

where the meson degrees of freedom are not explicit. The operator T measures the energy in the absence of the interaction operator U and is the sum of rest-mass and kinetic-energy operators. In terms of the nucleon creation operator $a_{\vec{p}_1}^\dagger$

$$|\Psi^{(+)}(\vec{p}_1,\vec{p}_2))=\left(a_{\vec{p}_1}^\dagger+\frac{1}{E_1+E_2-H+i\epsilon}\,j_{\vec{p}_1}^\dagger\right)|\Psi_{\vec{p}_2}), \tag{3.7}$$

where

$$j_{\vec{p}_1}^\dagger=[U,a_{\vec{p}_1}^\dagger] \tag{3.8}$$

and $|\Psi_{\vec{p}_2})$ is the target-state ket. The following relations will be useful later:

$$[T,a_{\vec{p}_1}^\dagger]=E_1a_{\vec{p}_1}^\dagger, \tag{3.9a}$$

$$j_{\vec{p}_1}^\dagger|\Psi_{\vec{p}_2})=(H-E_1-E_2)a_{\vec{p}_1}^\dagger|\Psi_{\vec{p}_2}), \tag{3.9b}$$

$$(\Psi_f^{(-)}|j_{\vec{p}_1}^\dagger=-(\Psi_f^{(-)}|a_{\vec{p}_1}^\dagger(H+E_1-E_f). \tag{3.9c}$$

Our first task is to obtain an expression for $\langle f|j_\pi^a(0)|p_1,p_2\rangle$ in terms of $F(q_0)$. For this purpose let us generalize the definition of Eq. (2.5) in the following manner

$$F(q_0)=\frac{i\sqrt{2}}{f_\pi}\int d^4x\,e^{iq_0x_0}(m_\pi^{\,2}-q_0^{\,2})\times\langle f|T[D^a(x),\bar{j}(0)]|p_2'\rangle u(p_1,s). \tag{3.10}$$

This differs from Eq. (2.5) in having an arbitrary momentum p_2' for the target. By definition

$$\vec{p}_1=\vec{p}_f-\vec{p}_2'$$

and (3.11)

$$E_1=p_{1_0}=(M^2+\vec{p}_1^{\,2})^{1/2}.$$

Note that $E_f+m_\pi\neq E_1+E_2'$, in general. The two quantities are equal when $p_2' = p_2$, the value used in the previous section. It is straightforward to verify that

$$F(E_2' + E_1 - E_f) = -\langle f | j_\pi^a(0) | p_1, p_2'(in)\rangle . \quad (3.12)$$

The right-hand side is the pion-production amplitude when $(p_f - p_1 - p_2')^2 = m_\pi^2$, which occurs when $p_2' = p_2$. By writing $\partial^2/\partial x_0^2$ for $-q_0^2$ and working out the derivatives of the time-ordered product one finds that

$$F(q_0) = \frac{\sqrt{2}}{f_\pi} q_0 \int d^4x \langle f | [D^a(x), \bar{j}(0)]\delta(x_0) | p_2'\rangle u(\vec{p}_1, s) + \frac{i\sqrt{2}}{f_\pi} \int d^4x \langle f | [\dot{D}^a(x), \bar{j}(0)]\delta(x_0) | p_2'\rangle u(\vec{p}_1, s)$$
$$+ i \int d^4x\, e^{iq_0x_0} \langle f | T[j_\pi^a(x), \bar{j}(0)] | p_2'\rangle u(\vec{p}_1, s) . \quad (3.13)$$

In a Lagrangian field theory which does not contain derivative couplings the equal-time commutator in the first term of the right-hand side above vanishes if we assume, in addition, that $(\sqrt{2}/f_\pi)D^a(x)$ is the canonical pion field (i.e., the strong version of PCAC). For simplicity we assume that the first term is zero here also. We note too that as $q_0 \to \infty$ only the second term (the seagull term) is nonvanishing. Denoting this term by $F(\infty)$ we write

$$F(q_0) = F(\infty) - (2\pi)^3 \Big[\sum_{|n\rangle} \langle f | j_\pi^a(0) | n\rangle\langle n | \bar{j}(0) | p_2'\rangle \frac{\delta^{(3)}(\vec{p}_f - \vec{p}_n)}{E_f + q_0 - E_n + i\epsilon}$$
$$- \sum_{|m\rangle} \langle f | \bar{j}(0) | m\rangle\langle m | j_\pi^a(0) | p_2'\rangle \frac{\delta^{(3)}(\vec{p}_m - \vec{p}_2')}{E_m + q_0 - E_2' + i\epsilon}\Big] u(\vec{p}_1, s) . \quad (3.14)$$

The states $|n\rangle$ have baryon number $A+1$ while the states $|m\rangle$ have baryon number A. We truncate these sums by retaining only the states with no pions since the denominators have much larger values for the other intermediate states. Furthermore, the denominator in the crossed term is smallest for $E_m \simeq E_2'$. For these terms we write

$$\delta^{(3)}(\vec{p}_m - \vec{p}_2')\langle m | j_\pi^a(0) | p_2'\rangle = \frac{\sqrt{2}}{f_\pi}[m_\pi^2 - (p_m - p_2')^2]\langle m | D^a(0) | p_2'\rangle \delta^{(3)}(\vec{p}_m - \vec{p}_2')$$
$$= -i\delta^{(3)}(\vec{p}_m - \vec{p}_2') \frac{\sqrt{2}}{f_\pi}[m_\pi^2 - (E_m - E_2')^2](E_m - E_2')\langle m | A_0^a(0) | p_2\rangle .$$

Since these terms are most important when $E_m \simeq E_2'$, we may replace A_0^a by the β-decay operator in the following way, writing

$$(2\pi)^3\delta^{(3)}(\vec{p}_m - \vec{p}_2')\langle m | j_\pi^a(0) | p_2'\rangle = \frac{-i\sqrt{2}m_\pi^2}{f_\pi}(\Psi_m | (H - E_2')(\Lambda^a - \mathfrak{A}^a) | \Psi_{\vec{p}_2'}) \quad (3.15)$$

and

$$\Lambda^a = g_A \sum_i \frac{\vec{\sigma}^i \cdot \vec{p}^i}{M} \tfrac{1}{2}\tau_i^a , \quad (3.16)$$

and the operator $\vec{p}^i$ measures the momentum of the ith nucleon in the pion rest frame. In the above equation we have introduced the operator $\mathfrak{A}^a$ in the following manner. Consider a state $|\Psi_n^{(\pm)})$. If it is a bound state then

$$\mathfrak{A}^a | \Psi_n) = (\Psi_n | A_0^a(0) | \Psi_n) | \Psi_n)$$

and for a positive-energy (continuum) state containing two or more particles (simple or composite)

$$\mathfrak{A}^a | \Psi_n^{(\pm)}) = \Big\{ \sum_i (f_i | A_0^a(0) | f_i) \Big\} | \Psi_n^{(\pm)}) ,$$

where the $|f_i)$'s are the states of the asymptotically separated constituents of $|\Psi_n^{(\pm)})$.

For the matrix elements $\langle n | \bar{j}(0) | p_2'\rangle$ and $\langle f | \bar{j}(0) | m\rangle$ we use the nonrelativistic counterparts

$$(2\pi)^3\delta^{(3)}(\vec{p}_n - \vec{p}_2' - \vec{p}_1)\langle n | \bar{j}(0) | p_2\rangle = (\Psi_n^{(-)} | j_{\vec{p}_1}^\dagger | \Psi_{\vec{p}_2'}) ,$$
$$(2\pi)^3\delta^{(3)}(\vec{p}_f - \vec{p}_m - \vec{p}_1)\langle f | \bar{j}(0) | m\rangle = (\Psi_f^{(-)} | j_{\vec{p}_1}^\dagger | \Psi_m^{(+)}) . \quad (3.17)$$

Finally, we introduce

$$\mathfrak{F}(q_0) = (2\pi)^3\delta^{(3)}(\vec{p}_f - \vec{p}_1 - \vec{p}_2')F(q_0) ,$$
$$\mathfrak{F}(\infty) = (2\pi)^3\delta^{(3)}(\vec{p}_f - \vec{p}_1 - \vec{p}_2')F(\infty) . \quad (3.18)$$

Using Eqs. (3.5), (3.15), and (3.17) we obtain

$$\mathfrak{F}(q_0) = \mathfrak{F}(\infty) - (\Psi_f^{(-)} | V_0^a \frac{1}{E_f + q_0 - H + i\epsilon} j_{\vec{p}_1}^\dagger | \Psi_{\vec{p}_2'}) + \frac{im_\pi^2\sqrt{2}}{f_\pi}(\Psi_f^{(-)} | j_{\vec{p}_1}^\dagger \frac{H - E_2'}{H + q_0 - E_2' + i\epsilon}(\Lambda^a - \mathfrak{A}^a) | \Psi_{\vec{p}_2'}) . \quad (3.19)$$

The unknown quantity $F(\infty)$ may be eliminated with the help of the known soft-pion limit. From Eqs. (2.8) and (2.7b) and from the vanishing of $q_0 M_0(q_0)$ in the limit that $q_0 \to 0$, we obtain with some rearrangement

$$F(0) = -\frac{i m_\pi{}^2\sqrt{2}}{f_\pi}[\langle f|[\bar{\psi}(0), \mathcal{Q}^a]|p_2'\rangle - \langle f|\bar{\psi}(0)|p_2'\rangle\gamma_5\tfrac{1}{2}\tau^a](E_f - E_1 - E_2')\gamma_0 u(\vec{p}_1, s). \tag{3.20}$$

Note that

$$-\bar{\psi}(0)\gamma_5\tfrac{1}{2}\tau^a = \left[\int d^3\vec{x}\,\bar{\psi}(\vec{x},0)\gamma_0\gamma_5\tfrac{1}{2}\tau^a\psi(\vec{x},0), \bar{\psi}(0)\right]. \tag{3.21}$$

The nonrelativistic form for the Hermitian operator in the commutator is Λ^a/g_A. Ignoring the difference between g_A and 1 we write

$$(2\pi)^3\delta^{(3)}(\vec{p}_f - \vec{p}_1 - \vec{p}_2')F(0) = \mathfrak{F}(0),$$

$$a = -\frac{i m_\pi{}^2\sqrt{2}}{f_\pi}(f|[\Lambda^a - \mathcal{Q}^a, a^\dagger_{\vec{p}_1}]|p_2')(E_f - E_1 - E_2'). \tag{3.22}$$

Eliminating $F(\infty)$ with the help of $F(0)$ we get

$$\mathfrak{F}(q_0) = \mathfrak{F}(0) - (\Psi_f^{(-)}|V_0^a\left(\frac{1}{E_f + q_0 - H + i\epsilon} - \frac{1}{E_f - H + i\epsilon}\right)j^\dagger_{\vec{p}_1}|\Psi_{\vec{p}_2'}) - \frac{i m_\pi{}^2\sqrt{2}}{f_\pi}q_0(\Psi_f^{(-)}|j^\dagger_{p_1}\frac{1}{H + q_0 - E_2' + i\epsilon}(\Lambda^a - \mathcal{Q}^a)|\Psi_{\vec{p}_2'}). \tag{3.23}$$

At this stage we use Eqs. (3.12) and (3.5) and find that

$$\begin{aligned}\mathfrak{F}(E_2' + E_1 - E_f) = {} & \mathfrak{F}(0) - (\Psi_f^{(-)}|V_0^a\left(\frac{1}{E_2' + E_1 - H + i\epsilon} - \frac{1}{E_f - H + i\epsilon}\right)j^\dagger_{\vec{p}_1}|\Psi_{\vec{p}_2'})\\ & - \frac{i m_\pi{}^2\sqrt{2}}{f_\pi}(\Psi_f^{(-)}|j^\dagger_{\vec{p}_1}\frac{1}{H + E_1 - E_f - i\epsilon}(\Lambda^a - \mathcal{Q}^a)|\Psi_{\vec{p}_2'})(E_2' + E_1 - E_f)\\ = {} & (\Psi_f^{(-)}|V_0^a|\Psi^{(+)}(\vec{p}_1, \vec{p}_2')).\end{aligned} \tag{3.24}$$

Using Eq. (3.7) for the initial state wave function we have

$$(\Psi_f^{(-)}|V_0^a|\Psi^{(+)}(\vec{p}_1, \vec{p}_2')) = (\Psi_f^{(-)}|V_0^a\left(a^\dagger_{\vec{p}_1} + \frac{1}{E_1 + E_2' - H + i\epsilon}j^\dagger_{\vec{p}_1}\right)|\Psi_{\vec{p}_2'}); \tag{3.25}$$

comparing the two expressions we obtain

$$\begin{aligned}(\Psi_f^{(-)}|V_0^a a^\dagger_{\vec{p}_1}|\Psi_{\vec{p}_2'}) = {} & -\mathfrak{F}(0) - (\Psi_f^{(-)}|V_0^a\frac{1}{E_f - H + i\epsilon}j^\dagger_{\vec{p}_1}|\Psi_{\vec{p}_2'})\\ & - \frac{i m_\pi{}^2\sqrt{2}}{f_\pi}(\Psi_f^{(-)}|j^\dagger_{\vec{p}_1}\frac{1}{H + E_1 - E_f - i\epsilon}(\Lambda^a - \mathcal{Q}^a)|\Psi_{\vec{p}_2'})(E_f - E_1 - E_2').\end{aligned} \tag{3.26}$$

Using Eqs. (3.9b) and (3.9c) to re-express the current operators in terms of creation operators we have

$$(\Psi_f^{(-)}|V_0^a\frac{1}{E_f - H + i\epsilon}a^\dagger_{\vec{p}_1}|\Psi_{\vec{p}_2'}) = \frac{i m_\pi{}^2\sqrt{2}}{f_\pi}(\Psi_f^{(-)}|\Lambda^a - \mathcal{Q}^a)a^\dagger_{\vec{p}_1}|\Psi_{\vec{p}_2'}). \tag{3.27}$$

This relation is true for arbitrary $|\Psi_{\vec{p}_2'})$. Since $a^\dagger_{\vec{p}_1}|\Psi_{\vec{p}_2'})$ form a complete set of states (though not necessarily an orthonormal basis) we conclude that

$$(\Psi_f^{(-)}|V_0^a = -\frac{i\sqrt{2}m_\pi{}^2}{f_\pi}(\Psi_f^{(-)}|(\Lambda^a - \mathcal{Q}^a)(E_f - H). \tag{3.28}$$

Finally, for the production amplitude we find

$$(\Psi_f^{(-)}|V_0^a|\Psi^{(+)}(\vec{p}_1, \vec{p}_2)) = -\frac{i\sqrt{2}m_\pi{}^3}{f_\pi}(\Psi_f^{(-)}|\Lambda^a|\Psi^{(+)}(\vec{p}_1, \vec{p}_2)). \tag{3.29}$$

Explicitly,

$$V_0^a(\vec{x}) = -\frac{i g_A\sqrt{2}m_\pi{}^3}{f_\pi}\sum_i \frac{\vec{\sigma}^i\cdot\vec{p}^i}{M}\tfrac{1}{2}\tau_i^a \tag{3.30}$$

and the physical amplitude is given to first order in V_0^a by

$$T^\pi(m_\pi) \simeq (\phi_f^{(-)}|V_0^a|\phi_i^{(+)}), \tag{3.31}$$

where $\phi_i^{(+)}$ and $\phi_f^{(-)}$ are the relative wave functions for the initial and final $(A+1)$-nucleon systems, respectively, satisfying the appropriate boundary conditions.

For arbitrary pion momenta we have after applying a Gallilean transformation

$$V_{\vec{k}}^a = -\frac{\sqrt{2}g_A m_\pi^2}{f_\pi}\sum_i e^{-i\vec{k}\cdot\vec{r}_i}\vec{\sigma}\cdot\left(-\frac{im_\pi\vec{\nabla}_i}{M}-\vec{k}\right)\tfrac{1}{2}\tau_i^a. \tag{3.32}$$

IV. DISCUSSION

The corrections to the soft-pion limit contained in equation (3.31) have a simple description. We write

$$\begin{aligned}T^\pi(m_\pi) &= (\phi_f^{(-)}|V_0^a|\phi_i^{(+)})\\ &= \frac{1}{m_\pi}(\phi_f^{(-)}|(E_i-E_f)V_0^a|\phi_i^{(+)})\\ &= \frac{1}{m_\pi}\{(\phi_f^{(-)}|-UV_0^a|\phi_i^{(+)})\\ &\qquad + (\phi_f^{(-)}|V_0^a(E_i-K)|\phi_i^{(+)})\},\end{aligned} \tag{4.1}$$

since at threshold K commutes with V_0^a by explicit construction. If one ignores the initial-state distortion and replaces $\phi_i^{(+)}$ by $\chi_p\phi_A$ in the above equation where χ_p is a plane-wave state for the relative motion and ϕ_A is the target ground state, the second term vanishes, leaving

$$\begin{aligned}T^\pi(m_\pi) &\simeq -\frac{1}{m_\pi}(\phi_f^{(-)}|UV_0^a|\chi_p\phi_A)\\ &\simeq -\frac{i\sqrt{2}m_\pi^3}{f_\pi}(\phi_f^{(-)}|U\Lambda^a|\chi_p\phi_A).\end{aligned} \tag{4.2}$$

Neglecting the emission of pions by the target, this becomes

$$T^\pi(m_\pi) \sim \frac{g_A m_\pi^2\sqrt{2}}{f_\pi}\sum_{s'}(\phi_f^{(-)}|U|\chi_{ps'}\phi_A) \times(\chi_{ps'}|\vec{\sigma}\cdot\left(\frac{-i\vec{\nabla}}{M}\right)\tfrac{1}{2}\tau^a|\chi_{ps}), \tag{4.3}$$

which is just the nonrelativistic limit of Eq. (2.15) above. Thus, to the extent that we ignore pion rescattering the soft- and "hard"-pion limits differ by the inclusion of the initial-state interaction and of the emission of pions by the target. Naturally these are very important mechanisms and must be included.

The form of the pion-production potential is also not surprising. If we substitute for the ratio g_A/f_π the corresponding quantity given by the Goldberger-Treiman relation

$$f_\pi = \sqrt{2}Mm_\pi^2 g_A/g_r$$

we obtain

$$V_{\vec{k}}^a = -\frac{m_\pi}{2M}g_r\sum_i e^{-i\vec{k}\cdot\vec{r}_i}\vec{\sigma}_i\left(-\frac{im_\pi}{M}\vec{\nabla}_i-\vec{k}\right)\tau_i^a,$$

which is just the potential which would be given by the pseudovector pion-nucleon coupling. This was not totally unexpected, certainly, since the substitution of the divergence of the axial-vector current for the pion field necessarily results in an effective pseudovector coupling. It is interesting, and this is an important result of the present work, that the form of this coupling is not altered by the dispersive corrections.

ACKNOWLEDGMENT

The authors wish to thank E. F. Redish and M. Rho for interesting comments. The presentation here has benefitted from the excellent review of M. Ericson and M. Rho,[9] which contains a careful discussion of our approach.

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*Work supported in part by the U.S. Energy Research and Development Administration.

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[10]We use throughout the Minkowski metric for four-vectors and Dirac matrices.

[11]See, for example, M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).