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ISOTENSOR ELECTROMAGNETIC CURRENTS AND NUCLEAR ISOBAR MASSES

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The effect of a possible isotensor $\gamma N\Delta$ coupling on the isobaric-multiplet mass equation is investigated. It is found that discrepancies between theory and experiment for the d-term cannot be explained by the contribution from isotensor electromagnetic currents.

There has been interest for a considerable time in discovering experimentally whether the photon has elementary couplings carrying an isospin different from 0 or 1. To date no such coupling has been definitely observed. The earliest proposal that a ($T = 2$) coupling of the photon was not excluded by data was put forward by Grishin et al. [1] and by Dombey and Kabir [2]. Since such a coupling cannot occur between nucleon states it was suggested by Shaw [3] that the reaction $\gamma N \to \pi N$ in the region of the $\Delta(1236)$ resonance would be the most likely place to search for an isotensor electromagnetic current. This idea was later elaborated by Sanda and Shaw [4] and by Donnachie and Shaw [5]. A resumé of this and other phenomena which would give evidence for an isotensor e.m. current may be found in the review of Donnachie [6].

The existence of an elementary isotensor e.m. interaction would be evidenced by several phenomena in nuclear physics as well. Those proposed to date are:

1) the presence of higher order terms than quadratic in the isobaric-multiplet mass equation [3, 7, 8],

2) the existence of ($T = 2$) electromagnetic transitions [2, 8],

3) a lack of symmetry in the decay widths for corresponding ($T = 3/2$) $\to$ ($T = 1/2$) e.m. transitions in mirror nuclei [9].

A summary of the evidence from these three phenomena in 1969 has been given by Blin-Stoyle [9]. More recently, an examination of the last two reactions has been made by Chemtob and Furui [10]. It is the first of these, the isobaric-multiplet mass equation, which we shall study in the present contribution.

The isobaric-multiplet mass equation [11]

$$M(A, T, T_z) = a - bT_z + cT_z^2 \qquad (1)$$

relates the masses of the $2T+1$ members of an isobaric multiplet‡. A general review has been given by Jänecke [12]. In general, the coefficients a, b, and c will depend on A, T, and any other quantum numbers characterising the multiplet. The equation above, comprising only constant, linear, and quadratic terms in T_z, results if the electromagnetic interaction between the nucleons is limited to one-photon exchange and to the neutron-proton mass difference. Terms of next higher order leading to

$$M(A, T, T_z) = a - bT_z + cT_z^2 - dT_z^3 + eT_z^4 \qquad (2)$$

can arise in either of two ways: 1) by two-photon exchange or 2) by the presence of an elementary ($T = 2$) coupling of the photon between the nucleon and other baryons, say, a ($T = 2$) $\gamma N\Delta$ coupling. In addition, cubic and quartic terms in T_z will occur only if these two mechanisms are between more than two nucleons. In general, the d-term in lowest order results from three-nucleon interactions. The e-term results from the interaction of at least four nucleons. To the extent that the actual ($T = 2$) coupling results from a radiative correction to the known ($T = 1$) $\gamma N\Delta$ coupling, we may call both of these mechanisms two-photon exchange. We shall, however, restrict this name to the first mechanism.

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‡ We use throughout the convention that the proton has $T_z = 1/2$.

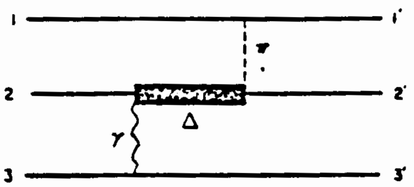

Fig. 1

Until a few years ago there was no evidence for the existence of terms of higher order than quadratic in the isobaric-multiplet mass equation [12]. Since that time considerable experimental effort has been made to test the validity of eq. (1). Complete and precise measurements of the masses are available now for fifteen isospin quartets ($T = 3/2$) and, recently, the measurement of the masses of all five members of a ($T = 2$) multiplet has been reported [13]. The data on isoquartets indicates that eq. (1) is remarkably well satisfied. In only one case, the $A = 9$ nuclei, is the d-term clearly different from zero. For this case d has the value 5.8 ± 1.5 keV [14].

A calculation for the contribution of two-photon exchange to the d-term of the $A = 9$ nuclei has been made by Bertsch and Kahana [15], who find a value $d = 3.6$ keV. (Of this 2.0 keV results from the inclusion of a charge-dependent term in the nuclear force [16]). There remain, then, perhaps 2 keV of the d-term which is not accounted for. It becomes reasonable, therefore, to examine to what extent this discrepancy could be explained by the as yet unobserved ($T = 2$) $\gamma N\Delta$ coupling.

The model which we have considered is shown in fig. 1. (A similar diagram involving four nucleons and two Δ's would give the lowest-order contribution of the isotensor current to the e-term). This is essentially the mechanism for a ($T = 2$) term in pion photoproduction proposed earlier by Shaw [3] but with a virtual photon and pion.

Assuming a pure M1 $\gamma N\Delta$ interaction the diagram of fig. 1 leads to a 3-body potential which is just the interaction of the isovector magnetic moment of nucleon 3 with the isotensor exchange magnetic moment of nucleons 1 and 2. This last operator has been calculated by Chemtob and Furui [10]. Borrowing their expression we arrive at the following 3-body potential for the d-term.

$$V_d(1,2,3) = -m_\pi(\mu_p - \mu_n)\frac{e^2}{4\pi}\left(\frac{m_\pi}{2M_N}\right)^2 g_{\pi NN}\xi V_4^{(3)}/16\sqrt{15}$$
$$\times \Sigma(1,2)\cdot\Sigma'(2,3)\frac{1}{|x_{23}|^3}\tau_3^{(1)}\tau_3^{(2)}\tau_3^{(3)} \quad (3)$$
$$+\text{all permutations of }(1,2,3)$$

with

$$\Sigma(1,2) = V_0(x_{12})(\sigma^{(1)} + \sigma^{(2)}) + V_2(x_{12})T^{(12)},$$

$$T^{(1,2)} = 3(\sigma^{(1)} + \sigma^{(2)})\cdot\hat{x}_{12}\hat{x}_{12} - (\sigma^{(1)} + \sigma^{(2)}),$$

$$\Sigma'(2,3) = 3(\sigma^{(3)}\cdot\hat{x}_{23})\hat{x}_{23} - \sigma^{(3)},$$

$$V_0(x) = \frac{\exp(-x)}{x}, \quad V_2(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)V_0(x),$$

$$x_{ij} = m_\pi(r_i - r_j)$$

$\xi = g_T/g_M$ is the ratio of the isotensor and isovector $\gamma N\Delta$ coupling constants and $V_4^{(3)}$ is the appropriate invariant amplitude for isovector excitation of the (3,3) resonance [17]. In units of m_π^{-3}, $V_4^{(3)}$ has the value 0.24. μ_p and μ_n are the magnetic moments of the proton and neutron in units of nuclear magnetons.

For a given ($T = 3/2$) multiplet the d-term is given by

$$6d = M(+3/2) - 3M(+1/2) + 3M(-1/2) - M(-3/2) \quad (4)$$

which leads to

$$d = -\frac{m_\pi}{\sqrt{15}}(\mu_p - \mu_n)\frac{e^2}{4\pi}\left(\frac{m_\pi}{2M_N}\right)^2 g_{\pi NN}\xi V_4^{(3)}$$
$$\times \sum_{i<j<k}\langle\Psi(A,T)|\Sigma(i,j)\cdot\Sigma'(j,k)\frac{1}{|x_{jk}|^3}|\Psi(A,T)\rangle. \quad (5)$$

For the $A = 9$ isoquartet the matrix element above nearly vanishes if we assume a pure $|P\rangle = |(1s_{1/2})^4 \times (1p_{3/2})^5\rangle$ configuration. Therefore the largest contribution will come from matrix elements connecting this to higher configurations, say, $|(1s_{1/2})^4(1p_{3/2})^4 \times (1d)\rangle$. We have generated an approximate D-state admixture in the ($A = 9$) wave function after the manner of Riska and Brown [18]

$$|D\rangle = -(1/E)\sum_{i<j} V_T(i,j)|P\rangle \quad (6)$$

where $V_T(i,j)$ is the two-body tensor force. We have tried to compensate for the fact that we are applying this method to the p-shell (rather than the s-shell) by reducing the effective energy denominator by one half. We have insured finiteness of our results by including a short-range correlation factor [18].

The numerical result is

$$d = (5\ \text{keV})\, g_T/g_M. \tag{7}$$

From several experiments [19–22] it is known that

$$|g_T/g_M| < 0.02. \tag{8}$$

Thus, the contribution of an isotensor $\gamma N\Delta$ coupling to the d-term cannot be more than 100 eV.

We have neglected in our calculation a possible E2 $\gamma N\Delta$ coupling. The isovector E2 transition multipole of the Δ is at least twenty times smaller than the corresponding M1 multipole [23]. This leads us to believe that the quadrupole moment of the Δ is very small. Hence, any possible isotensor E2 coupling constant of the Δ is likely correspondingly smaller than g_T and must, therefore, lead to an even smaller contribution to the d-term.

We conclude that the contribution of isotensor currents to the d-term is negligible.

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