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ANOMALOUS ENHANCEMENTS IN MULTIPLE-PION PRODUCTION WITH DEUTERONS

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The reaction $NN \to d\pi\pi$ is studied at intermediate energies in the Δ-isobar model. A large enhancement for low $\pi\pi$-masses in the isospin-singlet channel (ABC effect), the overall suppression of the isospin-triplet channel, and certain other features are predicted.

It has been known for some time that anomalous enhancements arise in multiple-pion production reactions with a light nucleus in the final state although a complete theory for these enhancements does not yet exist. Those reactions which have been studied experimentally are [1–5]:

$$p + n \to d + (\pi\pi)^0 \qquad (1)$$

$$p + p \to d + (\pi\pi)^+ \qquad (2)$$

$$p + d \to \tau + (\pi\pi)^0 \qquad (3)$$

$$p + d \to t + (\pi\pi)^+ . \qquad (4)$$

In these reactions for certain characteristic total energies a large enhancement (with respect to the three-particle phase space) is observed for the production of neutral pion pairs of very low effective mass (≈ 300 MeV). This enhancement is noticeably absent, however, in the analogous reactions for the production of charged pion pairs and the cross sections for these last reactions are suppressed overall as well. Thus, generally, this anomalous enhancement, or ABC effect after its discoverers [1], occurs only when the two pions are in a state of isospin zero.

In the reactions (3) and (4), which historically were studied first, this observation led to attempts [1, 6] to analyse the effect in terms of a low-energy pion-pion interaction, for which the proposed scattering length was between 1.0 and $3.0\, m_\pi^{-1}$. This was later contradicted by theoretical predictions [7] as well as by the experimental scattering length [8] disclosed by studies of the reaction

$$\pi^- + p \to n + \pi^0 + \pi^0 \qquad (5)$$

all of which were consistent with the much smaller value of $0.2\, m_\pi^{-1}$. The origin of this anomaly has still not been satisfactorily explained.

The theoretical analysis of reactions (1) and (2) is perhaps simpler than that of reactions (3) and (4). If we accept the contention [9] that the reaction

$$N + N \to N + N + \pi + \pi \qquad (6)$$

at intermediate energies proceeds predominantly through the production of two Δ's by one-pion exchange as shown in fig. 1a, then the natural Feynman diagram for the reaction

$$N + N \to d + \pi + \pi \qquad (7)$$

is that of fig. 1b. For simplicity taking all the particles to be scalar* we write this amplitude, apart from isospin, as

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* A more detailed calculation to be reported later, which assigned to the nucleon, deuteron, and Δ their proper spins, does not yield qualitatively different results.

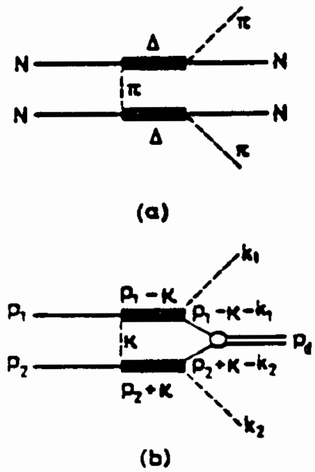

Fig. 1. Feynman diagrams for the reactions NN → NNππ and NN → dππ.

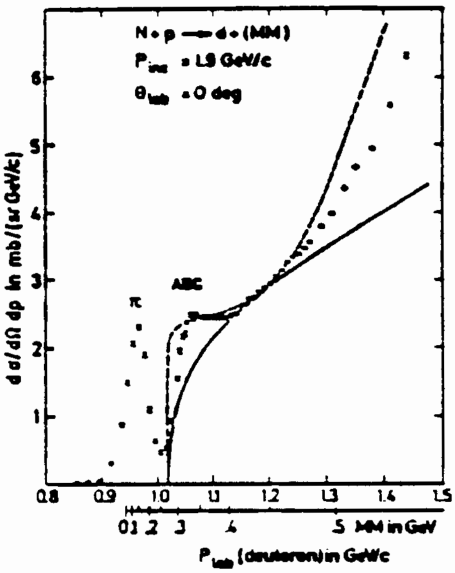

Fig. 2 The reaction np → dππ. The data of ref. [5] are compared with the theory (broken line) and Lorentz-invariant phase space (solid-line). The peak marked "π" corresponds to single-pion production. The abscissa gives the deuteron recoil momentum in the laboratory.

$$M(p_1,p_2,k_1,k_2) = \mathrm{i}\int \frac{d^4\kappa}{\kappa^2-m_\pi^2}\,\frac{A(Q_1{}^2)A(Q_2{}^2)}{(Q_1-k_1)^2-M_N^2}$$

$$\times\frac{\Gamma(Q_1-k_1,Q_2-k_2)}{(Q_2-k_2)^2-M_N{}^2}, \qquad (8)$$

where M_N and m_π are the masses of the nucleon and pion, respectively, and $\Gamma(Q_1-k_1,Q_2-k_2)$ is the vertex function for the deuteron. Q_1 and Q_2 are given by

$$Q_1 = p_1-\kappa,\; Q_2 = p_2+\kappa\,. \qquad (9)$$

$A(Q^2)$ is the amplitude for πN scattering through an intermediate Δ. Since all particles are scalar we assume that this quantity depends only on the total πN center-of-mass energy and not on the momentum transfer specifically. Writing this amplitude as $A(Q^2)$ determines its dependence off themass shell as well. Unless otherwise designated all momenta are 4-vectors..

Evaluating the isospin matrix elements explicitly, the amplitudes for the isospin-zero and isospin-one channels are given by

$$T(T=0) = (2/3)(M(p_1,p_2,k_1,k_2)+M(p_1,p_2,k_2,k_1)) \qquad (10a)$$

$$T(T=1) = (10/9\sqrt{6})(M(p_1,p_2,k_1,k_2)-M(p_1,p_2,k_2,k_1)). \qquad (10b)$$

In writing eq. (10) we have used the fact that the deuteron spatial wave function is symmetric and hence the amplitude is invariant under simultaneous exchange of initial nucleon and final pion momenta. Thus, after evaluating the isospin matrix elements it is sufficient that only the pion-spatial wave function have the proper symmetry. (A relation essentially identical to eq. (10) is obtained when the particle spins are treated properly.)

In the approximation that the deuteron binding energy is vanishingly small we have necessarily that

$$\frac{1}{P^2-M_N^2}\;\frac{1}{P'^2-M_N^2}\,\Gamma(P,P') \to g\,\delta^{(4)}(P-P') \qquad (11)$$

in terms of an effective coupling constant g. In this approximation M becomes

$$M(p_1,p_2,k_1,k_2) = \frac{\mathrm{i}g}{\kappa^2-m_\pi^2}A(Q_1{}^2)A(Q_2{}^2) \qquad (12)$$

where κ is now constrained to be $(p_1-p_2-k_1+k_2)/2$.

To obtain a realistic parameterization of $A(Q^2)$ we equate it with the $(3/2, 3/2^+)$ partial-wave amplitude for πN scattering. In terms of the decay momentum q,

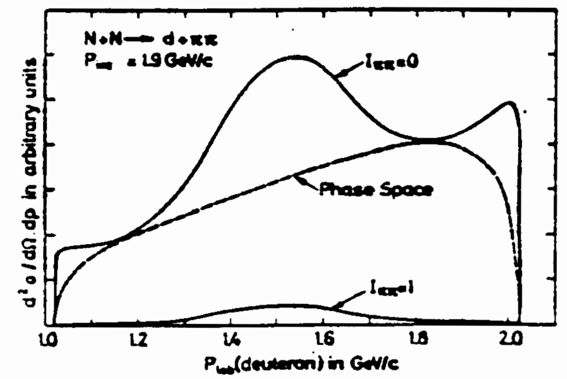

Fig. 3. Calculated deuteron recoil-momentum spectra in the laboratory for the isospin-zero and isospin-one channels for deuterons emitted at 0 deg. Note that in the laboratory all deuterons are emitted in the forward direction.

a scalar quantity, given by

$$q^2 = (4Q^2)^{-1}\,\lambda(Q^2, M_N^2, m_\pi^2) \qquad (13)$$

with $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, the πN scattering amplitude may be written [10]

$$A(Q^2) = \frac{\Gamma_\Delta(q)/q}{Q^2 - E_\Delta(q)^2}$$

$$E_\Delta(q) = M_\Delta + \mathrm{i}\,\Gamma_\Delta(q)/2$$

$$\Gamma_\Delta(q) = \gamma q \frac{R^2 q^2}{1 + R^2 q^2} \qquad (14)$$

with $M_\Delta = 1.24$ GeV, $R = 6.3(\mathrm{GeV}/c)^{-1}$, and $\gamma = 0.74$.

The differential cross section is given by

$$\mathrm{d}\sigma = \frac{1}{(2\pi)^5} f \frac{\mathrm{d}^3k_1}{2\omega_1} \frac{\mathrm{d}^3k_2}{2\omega_2} \frac{\mathrm{d}^3p_\mathrm{d}}{2E_\mathrm{d}} \delta^{(4)}(p_1 + p_2 - p_\mathrm{d} - k_1 - k_2)|T|^2 \qquad (15)$$

where $f = 1/(4E_1E_2|v_1 - v_2|)$ is the invariant flux factor. Integrating eq. (15) over the delta function one obtains, apart from a multiplicative factor, the differential cross section in the laboratory

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}|p_\mathrm{d}|\mathrm{d}\Omega_\mathrm{d}}\right)_\mathrm{lab} = \left.\frac{|p_\mathrm{d}|^2}{E_\mathrm{d}|p_1|}\right)_\mathrm{lab} \sqrt{\frac{M_{\pi\pi}{}^2 - 4m_\pi{}^2}{M_{\pi\pi}{}^2}} \int \mathrm{d}\Omega_k |T|^2 \qquad (16)$$

where $M_{\pi\pi} = \sqrt{(p_1 + p_2 - p_\mathrm{d})^2}$ is the effective mass of the two pions and Ω_k is the direction of the relative pion momentum in the $\pi\pi$-center-of-mass system.

The kinematic factor multiplying in the integral in eq. (16) is simply the two-body phase space. The cross section will be enhanced relative to this phase space when $|T|^2$ is large over the entire region of integration. This occurs when the two Δ's are simultaneously near resonance ($Q_1{}^2 \approx Q_2{}^2 \approx M_\Delta{}^2$) for all values of Ω_k. Taking the width of the Δ into account, this requires that the total center-of-mass energy satisfy the relation

$$E_{\mathrm{c.m.}} \gtrsim 2M_\Delta - \Gamma_\Delta$$

or, equivalently, that the incident nucleon momentum satisfy

$$|p_1|_\mathrm{lab} \gtrsim 1.7 \text{ GeV}/c. \qquad (17)$$

Data at incident momenta satisfying relation (17) are not very abundant. In order of appearance there are the data of Homer et al. [2] and Hall et al. [3] at 1.7 GeV/c, the earlier Saclay data [4] at 1.6, 1.8, and 1.9 GeV/c, and the most recent published Saclay data [5] at 1.9 GeV/c. This last satisfies best the above relation and has the highest statistical accuracy as well.

The predictions for the reaction np → dππ are compared with the most recent data of the Saclay deuteron group [5] in fig. 2. The calculation has simply been normalized to the data; no attempt has been made to treat the background. The theoretical predictions for the isospin-zero and isospin-one channels are shown in fig. 3 with consistent normalizations. Cancellations in the amplitude arising from the antisymmetry in the pion momenta lead to the suppression of the reaction pp → dππ seen by Hall et al. [3]. Fig. 3 shows the ABC enhancement for the deuteron emitted in both the forward and backward hemispheres in the center-of-mass system. We note that an enhancement is predicted for central values of the deuteron laboratory momentum (highest missing masses) as well.

The origin of these enhancements can be understood very simply*: Defining

$$K = k_1 + k_2\,, \quad k = k_1 - k_2\,, \qquad (18)$$

* This particular visualization is due to R. Lacaze and H. Navelet.

the squares of the invariant masses of the two Δ's may be written as

$$Q_1^2 = (p_d + K + k)^2/4\,, \quad Q_2^2 = (p_d + K - k)^2/4 \quad (19)$$

where p_d is the deuteron momentum. Since $K \cdot k = 0$, these two quantities will be equal (and $|T|^2$ will be large) over the whole region of integration of eq. (16) if and only if $p_d \cdot k = 0$ for all Ω_k. This condition can be satisfied in two ways:

1.) $k = 0$, in which case $K^2 = (2m_\pi)^2$.

2.) $p_d \cdot k = 0$ but $k \neq 0$. In order that this be true for all Ω_k we must have $p_d = 0$ in the rest frame of the two pions. In this case the pion-pion rest frame and the overall center-of-mass frame are the same and the entire kinetic energy of the system is taken up by the two pions whose effective mass, $\sqrt{K^2}$, will assume the largest possible value.

The low-mass enhancement ($k \approx 0$) corresponds to parallel decay of the two Δ's which have approximately the same invariant mass and little relative momentum; this enhancement is therefore expected to be more prominent when the total center-of-mass energy is near or slightly less than $2M_\Delta$ ($|p_1|_{lab} \lesssim 2.1$ GeV/c). The high-mass enhancement ($p_d \approx 0$ in the $\pi\pi$-rest frame), corresponding to anti-parallel decay of the two Δ's, is expected to be more prominent when $|p_1|_{lab} > 2.1$ GeV/c since, in this case, the Δ's must have relative momentum.

The ABC enhancements are seen clearly in all the data [2–5]. A high-mass enhancement is indicated in the data of Hall et al. [3] and also in recent unpublished data of the Saclay deuteron group [11] although it is much less pronounced than the central peak of fig. 3. However, the experimental situation with regard to the high-mass enhancement is unclear: reaction (1) has never been studied with a neutron beam but rather through the reaction

$$d + p \rightarrow p + d + (\pi\pi)^0 \quad (20)$$

assuming that the proton in the initial deuteron acts as a spectator particle. Fermi motion in the initial deuteron will broaden the experimental distributions and competing processes can introduce additional uncertainties. In particular, the data of fig. 2 are necessarily a mixture of reactions (1) and (2) although the contribution of the latter is not expected to be significant either from experimental evidence [3] or within the theory. Furthermore, it has not been suggested before that there is a high-mass enhancement associated with that at low mass and hence the high-missing-mass region has not attracted equal experimental attention. Only Hall et al. have obtained a complete momentum spectrum for the recoil deuteron.

The two-nucleon process has a natural extension to that for three nucleons, reactions (3) and (4). It remains to perform an explicit calculation but it should not be surprising to find many of the features of the simpler reaction reflected there. Experimentally they are even more pronounced [5]. The relative suppression of the production of two pions in a state of isospin one is well known [1]. In addition to the low-mass enhancements a high-mass enhancement is very prominent. In analogy with the simple case we would expect the ABC effect to be most pronounced when the total center-of-mass energy is close to $M_N + 2M_\Delta$, which corresponds to an incident deuteron laboratory momentum of 3.37 GeV/c. This is not inconsistent with what is currently known about the energy dependence of this effect [4].

Conclusion: the calculation presented here accounts for the known features of the ABC effect in the reaction NN → dππ. Certain other features, a high-mass enhancement and the dependence on total energy are predicted. Existing data are not sufficient to permit a detailed comparison of theory and experiment. We suggest that reaction (1) be studied directly with a neutron beam with momenta from 1.7 to 2.2 GeV/c and the entire recoil deuteron momentum spectrum be made the object of careful observation. With the neutron beam intensities now available such an experiment should be feasible.

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