```
      SUBROUTINE QUEST(NOBS,VREF,WOBS,DEV,QUIBBL,FIBBL,QUACC,NEWT,
     1                 IMETH,QOPT,COVAR,SIGTOT,RESIDU,TASTE,ICON,IOPT)
C
C-----VERSION DATE- JANUARY 19, 1987 ----------------------------------C
C                                                                      C
C-----FUNCTION-- QUEST COMPUTES THE QUATERNION OF THE OPTIMAL          C
C                ROTATION CARRYING A SET OF REFERENCE UNIT             C
C                VECTORS INTO A SET OF CORRESPONDING OBSERVATION       C
C                UNIT VECTORS AND THE COVARIANCE MATRIX OF THE         C
C                QUATERNION. THE ROTATION MINIMIZES A QUADRATIC        C
C                LOSS FUNCTION                                         C
C                                                                      C
C                ANALYSIS FOR THIS ALGORITHM IS GIVEN IN               C
C                   M. D. SHUSTER, APPROXIMATE ALGORITHMS FOR          C
C                   FAST OPTIMAL ATTITUDE COMPUTATION, PAPER NO.       C
C                   78-1249, AIAA GUIDANCE AND CONTROL CONFER-         C
C                   ENCE, PALO ALTO, CALIF., AUGUST 1978               C
C                                                                      C
C-----ARGUMENT LIST--                                                  C
C                                                                      C
C     NAME           TYPE I/O     DESCRIPTION                          C
C     ----           ---- ---     -----------                          C
C                                                                      C
C     NOBS           I*4   I      NUMBER OF OBSERVATIONS               C
C                                                                      C
C     VREF(3,NOBS)   R*4   I      REFERENCE UNIT VECTORS               C
C                                                                      C
C     WOBS(3,NOBS)   R*4   I      OBSERVATION UNIT VECTORS             C
C                                                                      C
C     DEV(NOBS)      R*4   I      STANDARD DEVIATIONS PER AXIS OF      C
C                                 UNIT VECTOR MEASUREMENTS (IN RAD)    C
C                                 QUEST WILL SET AN ERROR CODE (ICON=3) C
C                                 IF ANY OF THE STANDARD DEVIATIONS IN  C
C                                 THE COMPUTATION ARE SET EQUAL TO ZERO.C
C                                                                      C
C     QUIBBL         R*4   I      CRITICAL VALUE FOR TESTING QUALITY   C
C                                 OF QUATERNION COMPUTATION            C
C                                                                      C
C     FIBBL          R*4   I      CRITICAL VALUE FOR TESTING CONDITION  C
C                                 OF OBSERVABILITY MATRIX              C
C                                                                      C
C     QUACC          R*4   I      DESIRED ACCURACY IN COMPUTING        C
C                                 OVERLAP EIGENVALUE (ALAM)            C
C                                                                      C
C     NEWT           I*4   I      NUMBER OF NEWTON-RAPHSON ITERATIONS  C
C                                 IN QUATERNION COMPUTATION            C
C                                                                      C
C     IMETH          I*4   I      ALGORITHM SELECTION FLAG             C
C                                 (IF IMETH = 0, THE METHOD            C
C                                 OF SEQUENTIAL ROTATIONS              C
C                                 IS NOT IMPLEMENTED                   C
C                                                                      C
C     QOPT(4)        R*4   O      OPTIMAL QUATERNION (DEFAULT VALUE    C
C                                 = (999.,999.,999.,999.))             C
C                                                                      C
C     COVAR(3,3)     R*4   O      COVARIANCE MATRIX (UNITLESS)         C
C                                 (DEFAULT VALUE = 999. IN EVERY       C
C                                 ELEMENT)                             C
C                                                                      C
C     SIGTOT         R*4   O      TOTAL STANDARD DEVIATION OF          C
C                                 OBSERVATION MEASUREMENTS             C
C                                                                      C
```

```
C     RESIDU        R*4   O      OVERLAP DEFECT (=1.0 - OVERLAP)       C
C                                                                      C
C     TASTE         R*4   O      STATISTICAL PARAMETER (IF VALUES OF   C
C                                DEV(NVEC) ARE CORRECT, TASTE WILL     C
C                                POSSESS A CHI-SQUARE DISTRIBUTION     C
C                                WITH 2*NVEC DEGREES OF FREEDOM)       C
C                                                                      C
C     ICON          I*4   O      CONDITION CODE                        C
C                                 ICON=0 -- NO ERRORS OR POOR CONDI-   C
C                                           TIONS  DETECTED            C
C                                 ICON=1 -- OBSERVATION COUNT CHECK    C
C                                           (FEWER THAN TWO            C
C                                           OBSERVATIONS)              C
C                                 ICON=2 -- NEGATIVE WEIGHT CHECK      C
C                                           (NEGATIVE WEIGHT DETECTED) C
C                                 ICON=3 -- INFINITE WEIGHT CHECK      C
C                                           (AT LEAST ONE STANDARD     C
C                                           DEVIATION IS ZERO)         C
C                                 ICON=4 -- NORM CHECK                 C
C                                           (AT LEAST ONE REFERENCE    C
C                                           OR OBSERVATION VECTOR      C
C                                           HAS ZERO NORM)             C
C                                 ICON=5 -- OBSERVABILITY CHECK        C
C                                           (OBSERVABILITY MATRIX ILL- C
C                                           CONDITIONED)               C
C                                 ICON=6 -- FIRST QUATERNION FAILURE   C
C                                           CHECK (QUATERNION INACCU-  C
C                                           RATE AND IMETH=0)          C
C                                 ICON=7 -- SECOND QUATERNION FAILURE  C
C                                           CHECK (ALL FOUR QUATERNION C
C                                           COMPUTATIONS FAILED)       C
C                                                                      C
C     IOPT          I*4   O      OPTIMIZATION FLAG                     C
C                                 IOPT=0 -- QUATERNION COMPUTATION     C
C                                           SUCCEEDED ON FIRST COM-    C
C                                           PUTATION                   C
C                                                                      C
C                                 IOPT=1 -- A SUCCESSFUL QUATERNION    C
C                                           COMPUTATION WAS OBTAINED   C
C                                           BY METHOD OF SEQUENTIAL    C
C                                           ROTATIONS                  C
C                                 IOPT=2 -- ALL QUATERNION COMPUTATIONSC
C                                           FAILED. BEST QUATERNION IS C
C                                           OUTPUT.                    C
C                                                                      C
C-----EXTERNAL REFERENCES-- QUACOV - COMPUTES QUATERNION               C
C                                    COVARIANCE MATRIX                 C
C                                                                      C
C                           QUOMP  - COMPUTES UNNORMALIZED QUATERNION, C
C                                    OVERLAP, COMPUTATION QUALITY      C
C                                    PARAMETER                         C
C                                                                      C
C                           VEX    - COMPUTES A NORMALIZED VECTOR      C
C                                                                      C
C-----DESIGNER/PROGRAMMER--M. D. SHUSTER  C.S.C. 1/17/1979             C
C                                                                      C
C-----MODIFICATIONS-- F. L. MARKLEY GSFC CODE 552.2 1/19/87            C
C                     REMOVE LIMITATION ON NOBS.LT.10.  ALSO TEST FOR  C
C                     DEV**2=0 INSTEAD OF DEV=0 TO AVOID POSSIBLE      C
C                     REAL UNDERFLOW FOLLOWED BY DIVISION BY ZERO.     C
C                                                                      C
C-----Comment-- M. D. SHUSTER  4/03/2005                               C
C               Since 1981 IT HAS BEEN KNOW THAT, PROVIDED THE VALUES  C
C               OF DEV(NVEC) ARE CORRECT, TASTE WILL POSSESS A         C
C               CHI-SQUARE DISTRIBUTION WITH (2*NVEC-3) DEGREES OF     C
C               FREEDOM.                                               C
C                                                                      C
C               FURTHER INFORMATION ON THE QUEST ALGORITHM IS          C
```

```
C               AVAILABLE IN M. D. SHUSTER AND S. D. OH, THREE-AXIS    C
C               ATTITUDE DETERMINATION FROM VECTOR OBSERVATIONS,       C
C               JOURNAL OF GUIDANCE AND CONTROL, VOL. 4, NO. 1,        C
C               JAN-FEB 1981, PP. 70-77.                               C
C                                                                      C
C----------------------------------------------------------------------C
C                                                                      C
C                                                                      C
      REAL*8 V(3),W(3),SIG,SIG2,WT,QUALM,QNORM,DLAM,VNORM,WNORM,
     1       SIGTM2,SIGT2,SIGT,FIDDL,QUETCH,F(3,3),B(3,3),P(3,3),QUIP,
     2       Q(4),ALAM,QUAL,QQ(4,4),SALAM(4),SQUAL(4)
C
      DIMENSION VREF(3,*),WOBS(3,*),DEV(*),QOPT(4),COVAR(3,3)
C
C     PERFORM OBSERVATION COUNT CHECK
C
      IF(NOBS.LT.2) THEN
         ICON = 1
         GO TO 999
      ENDIF
C
C     SET DEFAULT VALUES FOR QUATERNION, COVARIANCE, CONDITION CODE,
C     AND OPTIMIZATION FLAG
C
      DO 20 I=1,3
      DO 10 J=1,3
      COVAR(I,J) = 999.
   10 CONTINUE
      QOPT(I) = 999.
   20 CONTINUE
      QOPT(4) = 999.
C
      ICON = 0
C
      IOPT = 0
C
C     CONVERT STANDARD DEVIATIONS TO REAL*8 AND PERFORM WEIGHT CHECKS.
C     IF OK, COMPUTE TOTAL STANDARD DEVIATION.  OTHERWISE,
C     SET CONDITION CODE AND RETURN.
C
      SIGTM2 = 0.D0
C
      DO 80 I=1,NOBS
      IF(DEV(I).LT.0.) THEN
         ICON = 2
         GO TO 999
      ENDIF
      SIG = DEV(I)
      SIG2 = SIG**2
      IF(SIG2.EQ.0.D0) THEN
         ICON = 3
         GO TO 999
      ENDIF
      SIGTM2 = SIGTM2 + 1.D0/SIG2
   80 CONTINUE
C
      SIGT2 = 1.D0/SIGTM2
      SIGT  = DSQRT(SIGT2)
      SIGTOT = SIGT
      FIDDL = FIBBL
      QUETCH = QUACC
      QUIP = QUIBBL
C
C     CONVERT VECTORS TO REAL*8 AND PERFORM NORM CHECKS.
C     IF OK COMPUTE WEIGHTS, OBSERVABILITY MATRIX, AND ATTITUDE
C     PROFILE MATRIX.  OTHERWISE, SET CONDITION CODE AND RETURN.
C
      DO 100 I=1,3
```

```
      DO  90 J=1,3
      F(I,J) = 0.D0
      B(I,J) = 0.D0
   90 CONTINUE
      F(I,I) = 1.D0
  100 CONTINUE
C
      DO 130 I=1,NOBS
      CALL VEX(VREF(1,I),VREF(2,I),VREF(3,I),V(1),V(2),V(3),VNORM)
      IF(VNORM.LE.0.D0) THEN
         ICON = 4
         GO TO 999
      ENDIF
      CALL VEX(WOBS(1,I),WOBS(2,I),WOBS(3,I),W(1),W(2),W(3),WNORM)
      IF(WNORM.LE.0.D0) THEN
         ICON = 4
         GO TO 999
      ENDIF
      SIG=DEV(I)
      WT = SIGT2/(SIG**2)
C
      DO 120 J=1,3
      DO 110 K=1,3
      F(J,K) = F(J,K) - WT*W(J)*W(K)
      B(J,K) = B(J,K) + WT*W(J)*V(K)
  110 CONTINUE
  120 CONTINUE
  130 CONTINUE
C
C
C     COMPUTE COVARIANCE MATRIX AND TEST PARAMETER
C
      CALL QUACOV(F,FIDDL,SIGT2,IDET,P)
C
C
C     TEST CONDITION OF OBSERVABILITY MATRIX. IF OK, CONTINUE,
C     OTHERWISE SET CONDITION CODE AND RETURN.
C
      IF(IDET.EQ.0)GO TO 140
      ICON = 5
      GO TO 999
C
C
C     COMPUTE UNNORMALIZED OPTIMAL QUATERNION AND TEST PARAMETER
C
  140 CALL QUOMP(B,NEWT,QUETCH,Q,ALAM,QUAL)
C
C
C     TEST QUATERNION COMPUTATION ACCURACY. IF OK, USE THIS FOR
C     COMPUTING THE NORMALIZED QUATERNION. OTHERWISE, KEEP TRYING
C
      IF(QUAL.GT.QUIP)GO TO 900
C
C
C     TEST WHETHER PROGRAM IS SPECIFIED TO USE METHOD OF SEQUENTIAL
C     ROTATIONS TO TRY TO FIND A BETTER RESULT. IF SO, IMPLEMENT
C     THIS METHOD. OTHERWISE, USE THE LAST COMPUTED QUATERNION.
C
      IF(IMETH.NE.0)GO TO 150
      ICON = 6
      GO TO 900
C
C
C     IMPLEMENT METHOD OF SEQUENTIAL ROTATIONS.  BY ROTATING REFERENCE
C     VECTORS AND PERFORMING THE CORRESPONDING INVERSE TRANSFORMATION
C     ON THE RESULTING QUATERNION, WE ATTEMPT TO OBTAIN A MORE ACCURATE
C     RESULT
C
```

```
C     LOAD FIRST QUATERNION, QUALITY PARAMETER, AND OVERLAP EIGENVALUE
C     FOR LATER REFERENCE.
C
  150 QQ(1,1) = Q(1)
      QQ(2,1) = Q(2)
      QQ(3,1) = Q(3)
      QQ(4,1) = Q(4)
C
      SALAM(1) = ALAM
      SQUAL(1) = QUAL
C
C
C     SET OPTIMIZATION FLAG TO SHOW THAT METHOD OF SEQUENTIAL ROTATIONS
C     IS BEING IMPLEMENTED.
C
      IOPT = 1
C
C     FIRST ROTATION.  UNNORMALIZED QUATERNION IS COMPUTED FOR A ROTA-
C     TION OF THE REFERENCE VECTORS THROUGH 180. DEGREES ABOUT THE
C     X-AXIS.  THE FOLLOWING STEPS ARE PERFORMED:
C
C     A NEW MEASUREMENT PROFILE MATRIX IS COMPUTED
C
      DO 160 I=1,3
      B(I,2) = -B(I,2)
      B(I,3) = -B(I,3)
  160 CONTINUE
C
C     THE UNNORMALIZED QUATERNION IS RECOMPUTED FOR THIS NEW VECTOR SET
C
      CALL QUOMP(B,NEWT,QUETCH,Q,ALAM,QUAL)
C
C     THE INVERSE TRANSFORMATION IS APPLIED TO THE QUATERNION, WHICH IS
C     LOADED IN THE SECOND COLUMN OF THE ARRAY QQ. LIKEWISE FOR THE
C     OTHER PARAMETERS.
C
      QQ(1,2) =  Q(4)
      QQ(2,2) = -Q(3)
      QQ(3,2) =  Q(2)
      QQ(4,2) = -Q(1)
C
      SALAM(2) = ALAM
      SQUAL(2) = QUAL
C
C     NOW WE TEST THIS QUATERNION FOR ACCURACY. IF IT IS OK, WE WILL
C     SET A FLAG TO DENOTE THAT THE SECOND COLUMN OF QQ WILL BE USED
C     TO COMPUTE THE NORMALIZED OPTIMAL QUATERNION AND PROCEED TO
C     THAT PART OF THE COMPUTATION. OTHERWISE, WE TRY THE NEXT
C     ROTATION.
C
      IF(QUAL.LT.QUIP)GO TO 170
C
      IQ = 2
      GO TO 800
C
C
C     WE NOW TRY A ROTATION OF THE REFERENCE VECTORS THROUGH 180. DEG
C     ABOUT THE Y-AXIS AND REPEAT THE SAME STEPS. NOTE THAT IN COMPUT-
C     ING THE NEW MEASUREMENT PROFILE MATRIX WE MUST TAKE ACCOUNT OF
C     PREVIOUS ROTATIONS.
C
  170 DO 180 I=1,3
      B(I,1) = -B(I,1)
      B(I,2) = -B(I,2)
  180 CONTINUE
C
      CALL QUOMP(B,NEWT,QUETCH,Q,ALAM,QUAL)
C
```

```
      QQ(1,3) =  Q(3)
      QQ(2,3) =  Q(4)
      QQ(3,3) = -Q(1)
      QQ(4,3) = -Q(2)
C
      SALAM(3) = ALAM
      SQUAL(3) = QUAL
C
      IF(QUAL.LT.QUIP)GO TO 190
C
      IQ = 3
      GO TO 800
C
C
C     NOW WE TRY A ROTATION OF THE REFERENCE VECTORS ABOUT THE Z-AXIS
C
  190 DO 200 I=1,3
      B(I,2) = -B(I,2)
      B(I,3) = -B(I,3)
  200 CONTINUE
C
      CALL QUOMP(B,NEWT,QUETCH,Q,ALAM,QUAL)
C
      QQ(1,4) = -Q(2)
      QQ(2,4) =  Q(1)
      QQ(3,4) =  Q(4)
      QQ(4,4) = -Q(3)
C
      SALAM(4) = ALAM
      SQUAL(4) = QUAL
C
      IF(QUAL.LT.QUIP)GO TO 210
C
      IQ = 4
      GO TO 800
C
C
C     STATEMENT 210 IS REACHED IF THE METHOD OF SEQUENTIAL ROTATIONS
C     FAILED TO FIND AN ACCEPTABLE QUATERNION. IF THIS IS A FREQUENT
C     OCCURENCE, IT PROBABLY INDICATES THAT QUACC, NEWT, AND QUIBBL
C     HAVE NOT BEEN CHOSEN PROPERLY. QUEST NOW SEARCHES FOR THE BEST
C     QUATERNION AND SETS THE OPIMIZATION FLAG TO 2.
C
  210 IOPT = 2
      ICON = 7
      IQ = 1
      QUALM = 0.D0
C
      DO 220 I=1,4
      IF(SQUAL(I).GT.QUALM) IQ = I
      IF(IQ.EQ.I) QUALM = SQUAL(I)
  220 CONTINUE
C
C
C     CHOOSE BEST QUATERNION AND OVERLAP EIGENVALUE
C
  800 DO 230 I=1,4
      Q(I) = QQ(I,IQ)
  230 CONTINUE
C
      ALAM = SALAM(IQ)
C
  900 DLAM = 1.D0 - ALAM
C
      QNORM = DSQRT(Q(1)**2 + Q(2)**2 + Q(3)**2 + Q(4)**2)
C
C     RENORMALIZE QUATERNION. CONVERT OUTPUT ARRAYS TO REAL*4
C
```

```
      DO 920 I=1,3
      DO 910 J=1,3
      COVAR(I,J) = P(I,J)
  910 CONTINUE
      QOPT(I) = Q(I)/QNORM
  920 CONTINUE
      QOPT(4) = Q(4)/QNORM
C
C     COMPUTE QUALITY PARAMETERS
C
      RESIDU = DLAM
C
      TASTE =  2.D0*DLAM/SIGT2
C
C
  999 RETURN
      END



C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =



      SUBROUTINE QUOMP(B,SIGTM2,NEWT,QUETCH,Q,ALAM,QUAL)
C
C-----VERSION DATE- JUNE 26, 1990 -------------------------------------C
C                                                                      C
C-----FUNCTION-- QUOMP COMPUTES AN UNNORMALIZED QUATERNION FOR THE     C
C               OPTIMAL ROTATION FROM THE NON-NORMALIZED ATTITUDE      C
C               PROFILE MATRIX                                         C
C                                                                      C
C               ANALYSIS FOR THIS ALGORITHM IS GIVEN IN                C
C                  M. D. SHUSTER, APPROXIMATE ALGORITHMS FOR           C
C                  FAST OPTIMAL ATTITUDE COMPUTATION, PAPER NO.        C
C                  78-1249, AIAA GUIDANCE AND CONTROL CONFER-          C
C                  ENCE, PALO ALTO, CALIF., AUGUST 1978                C
C                                                                      C
C-----ARGUMENT LIST--                                                  C
C                                                                      C
C     NAME          TYPE I/O     DESCRIPTION                           C
C     -----         ---- ---     -----------                           C
C                                                                      C
C     B(3,3)        R*8   I      NON-NORMALIZED ATTITUDE PROFILE       C
C                                MATRIX                                C
C                                                                      C
C     SIGTM2        R*8   I      SUMMATION OF INVERSE SQUARE OF THE    C
C                                STANDARD DEVIATIONS PER AXIS OF UNIT  C
C                                VECTOR MEASUREMENTS                   C
C                                                                      C
C     NEWT          I*4   I      NUMBER OF NEWTON-RAPHSON ITERATIONS   C
C                                                                      C
C     QUETCH        R*8   I      DESIRED ACCURACY IN DETERMINING       C
C                                OVERLAP EIGENVALUE (ALAM)             C
C                                                                      C
C     Q(4)          R*8   O      OUTPUT QUATERNION (UNNORMALIZED)      C
C                                                                      C
C     ALAM          R*8   O      OVERLAP EIGENVALUE (IF NEWT=0,        C
C                                THEN ALAM=1.D0)                       C
C                                                                      C
C     QUAL          R*8   O      QUAL = DABS(Q(4)) IS A MEASURE OF     C
C                                THE ACCURACY IN COMPUTING Q. QUAL     C
C                                IS LARGER THE MORE ACCURATE IS Q.     C
C                                                                      C
C-----EXTERNAL REFERENCES--NONE                                        C
C                                                                      C
C-----CALLED FROM--BQUEST                                              C
C                                                                      C
```

```
C-----DESIGNER/PROGRAMMER--M. D. SHUSTER  C.S.C. 1/17/1979             C
C                                                                      C
C-----MODIFICATIONS-- D. G. KUBITSCHEK  JHU-APL  06/25/90              C
C                     DECLARE ALL VARIABLES REAL*8. ALLOW FOR INPUT    C
C                     OF NON-NORMALIZED ATTITUDE PROFILE MATRIX.       C
C                                                                      C
C----------------------------------------------------------------------C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION B(3,3),Q(4),S(3,3),Z(3),SZ(3),SSZ(3)
C
C
      DO 6 I=1,3
      DO 5 J=1,3
      B(I,J)=B(I,J)/SIGTM2
    5 CONTINUE
    6 CONTINUE
C
C
C
C     COMPUTE MATRIX S = B + TRANSPOSE
C
C
      DO 20 I=1,3
      DO 10 J=1,3
      S(I,J) = B(I,J) + B(J,I)
   10 CONTINUE
   20 CONTINUE
C
C
C
C
C     COMPUTE TRACE, TRACE(ADJOINT), AND DETERMINANT OF S
C
      TRS = S(1,1)+S(2,2)+S(3,3)
C
      TRADJS = S(1,1)*S(2,2)+S(2,2)*S(3,3)+S(3,3)*S(1,1)
     1        -S(1,2)*S(2,1)-S(2,3)*S(3,2)-S(3,1)*S(1,3)
C
      DETS = S(1,1)*(S(2,2)*S(3,3)-S(2,3)*S(3,2))
     1      +S(1,2)*(S(2,3)*S(3,1)-S(2,1)*S(3,3))
     2      +S(1,3)*(S(3,2)*S(2,1)-S(3,1)*S(2,2))
C
C
      SIGMA = TRS/2.D0
C
C
C     COMPUTE VECTOR Z
C
      Z(1) = B(2,3) - B(3,2)
      Z(2) = B(3,1) - B(1,3)
      Z(3) = B(1,2) - B(2,1)
C
C
C     COMPUTE VECTORS SZ AND SSZ
C
      SZ(1) = S(1,1)*Z(1) + S(1,2)*Z(2) + S(1,3)*Z(3)
      SZ(2) = S(2,1)*Z(1) + S(2,2)*Z(2) + S(2,3)*Z(3)
      SZ(3) = S(3,1)*Z(1) + S(3,2)*Z(2) + S(3,3)*Z(3)
C
      SSZ(1) = S(1,1)*SZ(1) + S(1,2)*SZ(2) + S(1,3)*SZ(3)
      SSZ(2) = S(2,1)*SZ(1) + S(2,2)*SZ(2) + S(2,3)*SZ(3)
      SSZ(3) = S(3,1)*SZ(1) + S(3,2)*SZ(2) + S(3,3)*SZ(3)
C
C
C     SET ALAM (CALLED X) TO UNPERTURBED VALUE (PERFECT OVERLAP)
C
```

```
      X = 1.D0
C
C
C     IF NEWT IS GREATER THAN ZERO, COMPUTE NEW VALUE OF ALAM
C     USING NEWTON-RAPHSON METHOD NEWT TIMES. OTHERWISE, PROCEED
C     DIRECTLY TO THE COMPUTATION OF THE QUATERNION USING THE
C     STARTING VALUE FOR ALAM.
C
      IF(NEWT.LE.0)GO TO 40
C
C     DETERMINE COEFFICIENTS OF CHARACTERISTC POLYNOMIAL FOR ALAM
C                X**4 + D1*X**2 + D2*X + D3
C
      B1 = SIGMA**2 - TRADJS
      B2 = SIGMA**2 + Z(1)*Z(1) + Z(2)*Z(2) + Z(3)*Z(3)
      B3 = DETS + Z(1)*SZ(1) + Z(2)*SZ(2) + Z(3)*SZ(3)
      B4 = Z(1)*SSZ(1) + Z(2)*SSZ(2) + Z(3)*SSZ(3)
C
      D1 = -B1-B2
      D2 = -B3
      D3 = B1*B2+SIGMA*B3-B4
C
C     PERFORM NEWTON-RAPHSON ITERATIONS
C
      DO 30 I=1,NEWT
      X2 = X*X
      DX = -(((X2+D1)*X +D2)*X+D3)/((4.D0*X2+2.D0*D1)*X+D2)
      X = X+DX
C
C     TEST IF ALAM COMPUTATION EXCEEDS DESIRED ACCURACY.
C     IF SO, EXIT FROM LOOP
C
      IF(DABS(DX).LT.QUETCH)GO TO 40
C
   30 CONTINUE
C
C
C     COMPUTE UNNORMALIZED AXIS VECTOR
C
   40 ALPHA = X*X - SIGMA**2 + TRADJS
      BETA = X - SIGMA
      GAMMA = ALPHA*(X+SIGMA)-DETS
C
C     COMPUTE UNNORMALIZED QUATERNION
C
      Q(1) = ALPHA*Z(1) + BETA*SZ(1) + SSZ(1)
      Q(2) = ALPHA*Z(2) + BETA*SZ(2) + SSZ(2)
      Q(3) = ALPHA*Z(3) + BETA*SZ(3) + SSZ(3)
      Q(4) = GAMMA
C
C
C     SET VALUE OF OVERLAP EIGENVALUE AND QUALITY PARAMETER
C
      ALAM = X
C
      QUAL = DABS(GAMMA)
C
      DO 60 I=1,3
      DO 50 J=1,3
      B(I,J) = B(I,J)*SIGTM2
   50 CONTINUE
   60 CONTINUE
C
      RETURN
      END
```

```
C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =


      SUBROUTINE QUACOV(F,FIBBL,VARTOT,IDET,P)
C
C-----VERSION DATE- JUNE 25, 1990 ------------------------------------C
C                                                                     C
C-----FUNCTION-- QUACOV COMPUTES THE COVARIANCE MATRIX OF THE QUA-     C
C               TERNION COMPUTED BY QUEST                             C
C                                                                     C
C-----ARGUMENT LIST--                                                 C
C                                                                     C
C     NAME          TYPE I/O     DESCRIPTION                          C
C     ----          ---- ---     -----------                          C
C                                                                     C
C     F(3,3)        R*8   I      OBSERVABILITY MATRIX                 C
C                                                                     C
C     FIBBL         R*8   I      CONDITION LIMIT ON F                 C
C                                (IF DABS(DET F) IS LESS THAN FIDDL,  C
C                                THEN F IS ILL-CONDITIONED            C
C                                                                     C
C     VARTOT        R*8   I      TOTAL VARIANCE OF THE OBSERVATION    C
C                                VECTOR MEASUREMENTS (IN RAD**2)      C
C                                                                     C
C     IDET          I*4   O      CONDITION CODE                       C
C                                (IDET=0,IF F IS WELL-CONDITIONED     C
C                                (IDET=1,IF F IS ILL-CONDITIONED      C
C                                                                     C
C     P(3,3)        R*8   O      COVARIANCE MATRIX (UNITLESS)         C
C                                (DEFAULT VALUE = 999. IN EVERY       C
C                                ELEMENT)                             C
C                                                                     C
C-----EXTERNAL REFERENCES--NONE                                       C
C                                                                     C
C-----CALLED FROM--BQUEST                                             C
C                                                                     C
C-----DESIGNER/PROGRAMMER--M. D. SHUSTER  C.S.C. 1/17/1979            C
C                                                                     C
C-----MODIFICATIONS-- D. G. KUBITSCHEK  JHU-APL  06/26/90             C
C                     DECLARE ALL VARIABLES REAL*8.                   C
C                                                                     C
C---------------------------------------------------------------------C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION F(3,3),P(3,3)
C
C
C     SET INITIAL VALUE OF IDET
C
      IDET = 0
C
C
C     COMPUTE DETERMINANT OF F
C
C
      DETF = F(1,1)*(F(2,2)*F(3,3)-F(2,3)*F(3,2))
     1      +F(1,2)*(F(2,3)*F(3,1)-F(2,1)*F(3,3))
     2      +F(1,3)*(F(3,2)*F(2,1)-F(3,1)*F(2,2))
C
C
C     TEST CONDITION OF F. IF F IS ILL-CONDITIONED, SET CONDITION
C     CODE, COMPUTE DEFAULT COVARIANCE MATRIX, AND RETURN. OTHER-
C     WISE, CONTINUE.
C
```

```
      IF(DABS(DETF).GT.FIBBL)GO TO 50
C
      IDET = 1
C
      DO 40 I=1,3
      DO 30 J=1,3
      P(I,J) = 999.D0
   30 CONTINUE
   40 CONTINUE
C
      GO TO 999
C
C
   50 FAC = VARTOT/(4.D0*DETF)
C
C
C     COMPUTE COVARIANCE MATRIX
C
      P(1,1) = FAC*(F(2,2)*F(3,3)-F(2,3)*F(3,2))
      P(1,2) = FAC*(F(1,3)*F(3,2)-F(1,2)*F(3,3))
      P(1,3) = FAC*(F(1,2)*F(2,3)-F(1,3)*F(2,2))
      P(2,1) = FAC*(F(2,3)*F(3,1)-F(2,1)*F(3,3))
      P(2,2) = FAC*(F(1,1)*F(3,3)-F(1,3)*F(3,1))
      P(2,3) = FAC*(F(2,1)*F(1,3)-F(2,3)*F(1,1))
      P(3,1) = FAC*(F(3,2)*F(2,1)-F(3,1)*F(2,2))
      P(3,2) = FAC*(F(3,1)*F(1,2)-F(1,1)*F(3,2))
      P(3,3) = FAC*(F(1,1)*F(2,2)-F(1,2)*F(2,1))
C
C
  999 RETURN
      END


C = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = = =


      SUBROUTINE VEX(V1,V2,V3,DV1,DV2,DV3,DNORM)
C
C-----VERSION DATE- JUNE 25, 1990 -------------------------------------C
C                                                                      C
C-----FUNCTION-- VEX NORMALIZES A GIVEN VECTOR. THE NULL VECTOR        C
C                IS RETURNED AS THE NULL VECTOR.                       C
C                                                                      C
C-----ARGUMENT LIST--                                                  C
C                                                                      C
C     NAME          TYPE I/O      DESCRIPTION                          C
C     ----          ---- ---     -----------                           C
C                                                                      C
C     V1,V2,V3      R*8   I      COMPONENTS OF THE INPUT VECTOR        C
C                                                                      C
C     DV1,DV2,DV3   R*8   O      COMPONENTS OF THE OUTPUT VECTOR       C
C                                                                      C
C     DNORM         R*8   O      NORM OF THE INPUT VECTOR              C
C                                                                      C
C-----EXTERNAL REFERENCES--NONE                                        C
C                                                                      C
C-----CALLED FROM--INQUEST                                             C
C                                                                      C
C-----DESIGNER/PROGRAMMER--M. D. SHUSTER  C.S.C.  6/20/79              C
C                                                                      C
C-----MODIFICATIONS-- D. G. KUBITSCHEK  JHU-APL  06/25/90              C
C                     DECLARE ALL VARIABLES REAL*8                     C
C                                                                      C
C----------------------------------------------------------------------C
C
      REAL*8 DV1,DV2,DV3,DNORM
```

```
C
      DV1 = V1
      DV2 = V2
      DV3 = V3
C
      DNORM = DSQRT(DV1**2 + DV2**2 + DV3**2)
C
      IF(DNORM.EQ.0.D0)GO TO 999
C
      DV1 = DV1/DNORM
      DV2 = DV2/DNORM
      DV3 = DV3/DNORM
  999 RETURN
      END
```