Problem 65-1: A Least Squares Estimate of Satellite Attitude.

Grace Wahba

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PROBLEMS AND SOLUTIONS

EDITED BY MURRAY S. KLAMKIN

All problems and solutions should be sent to Murray S. Klamkin, Ford Scientific Laboratory, Mathematical and Theoretical Sciences, P.O. Box 2053, Dearborn, Michigan 48121, and should be submitted in accordance with the instructions given on the inside front cover. An asterisk placed beside a problem number indicates that the problem was submitted without solution. Proposers and solvers whose solution is published will receive 10 reprints of the corresponding problem section. Other solvers will receive just one reprint.

Problem 65–1, A Least Squares Estimate of Satellite Attitude, by GRACE WAHBA (International Business Machines Corporation).

Given two sets of n points $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$, and $\{\mathbf{v}_1^*, \mathbf{v}_2^*, \cdots, \mathbf{v}_n^*\}$, where $n \geqq 2$, find the rotation matrix M (i.e., the orthogonal matrix with determinant $+1$) which brings the first set into the best least squares coincidence with the second. That is, find M which minimizes

$$\sum_{j=1}^{n} \| \mathbf{v}_j^* - M\mathbf{v}_j \|^2.$$

This problem has arisen in the estimation of the attitude of a satellite by using direction cosines $\{\mathbf{v}_k^*\}$ of objects as observed in a satellite fixed frame of reference and direction cosines $\{\mathbf{v}_k\}$ of the same objects in a known frame of reference. M is then a least squares estimate of the rotation matrix which carries the known frame of reference into the satellite fixed frame of reference.

Problem 65–2, A Third Order Differential Equation, by DONALD E. AMOS (Sandia Corporation).

The differential equation

$$[D^3 + t^2D + 3t]\, y = 0$$

arises in a problem describing the motion of a particle in a magnetic field.
(1) Identify the power series solutions in terms of special functions,
(2) evaluate the associated integral

$$\int_0^t xy(x)\, dx,$$

and
(3) find asymptotic expressions for large t in (1) and (2).

Problem 65–3, On the Zeros of a Set of Polynomials, by M. N. S. SWAMY (University of Saskatchewan, Regina, Canada).

The polynomials $B_n(x)$ and $b_n(x)$ defined by the following relations appear in